COMP9517
Computer Vision

2023 Term 2 Week 1

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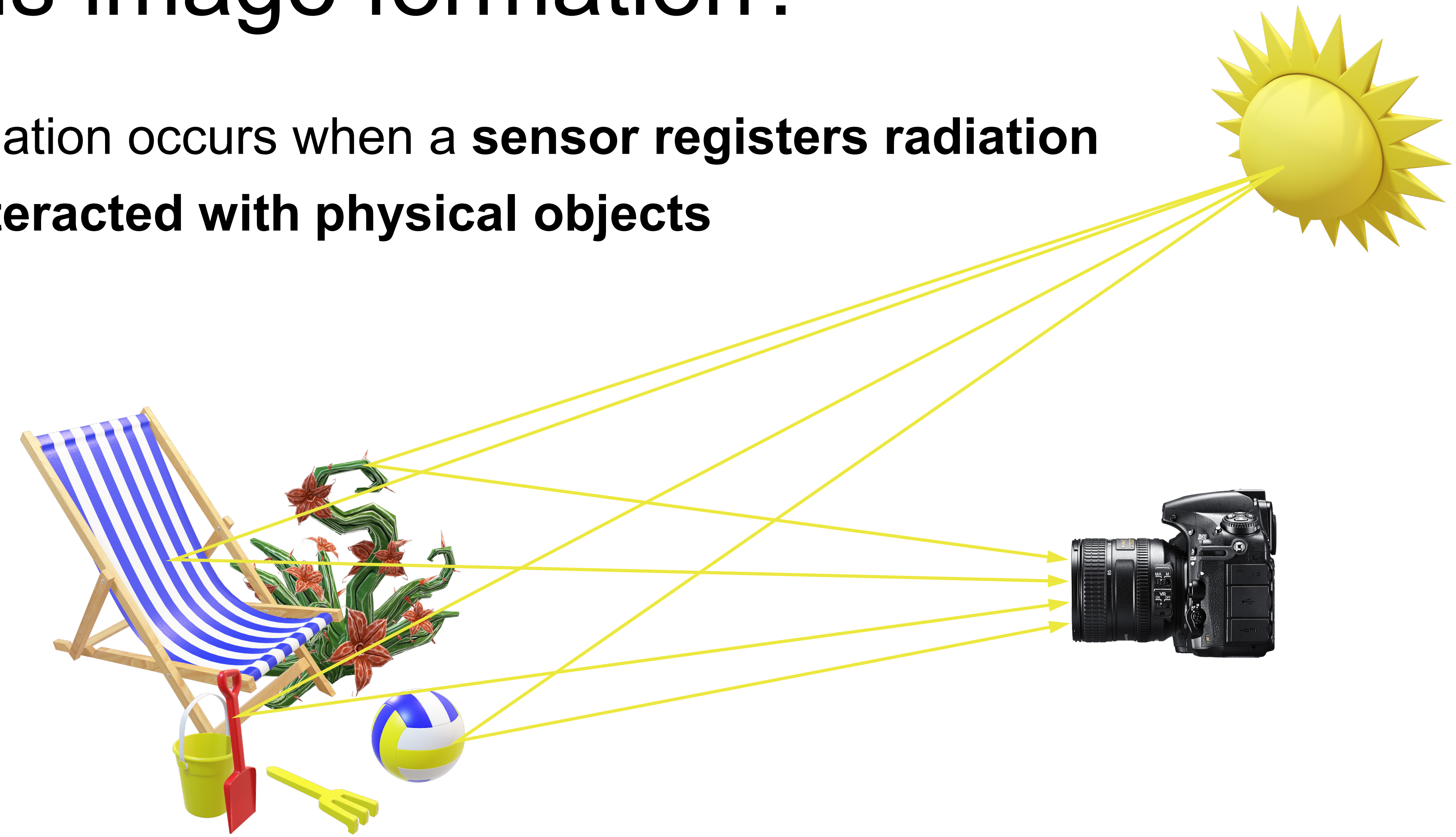
UNSW SYDNEY

Image Formation

What is image formation?

Image formation occurs when a **sensor registers radiation** that has **interacted with physical objects**



Geometry of image formation

Mapping world coordinates (3D) to image coordinates (2D)

- Pinhole camera model
- Projective geometry
- Projection matrix

Image formation

Idea 1: Put a piece of film in front of an object

Do we get a reasonable image?

Image formation

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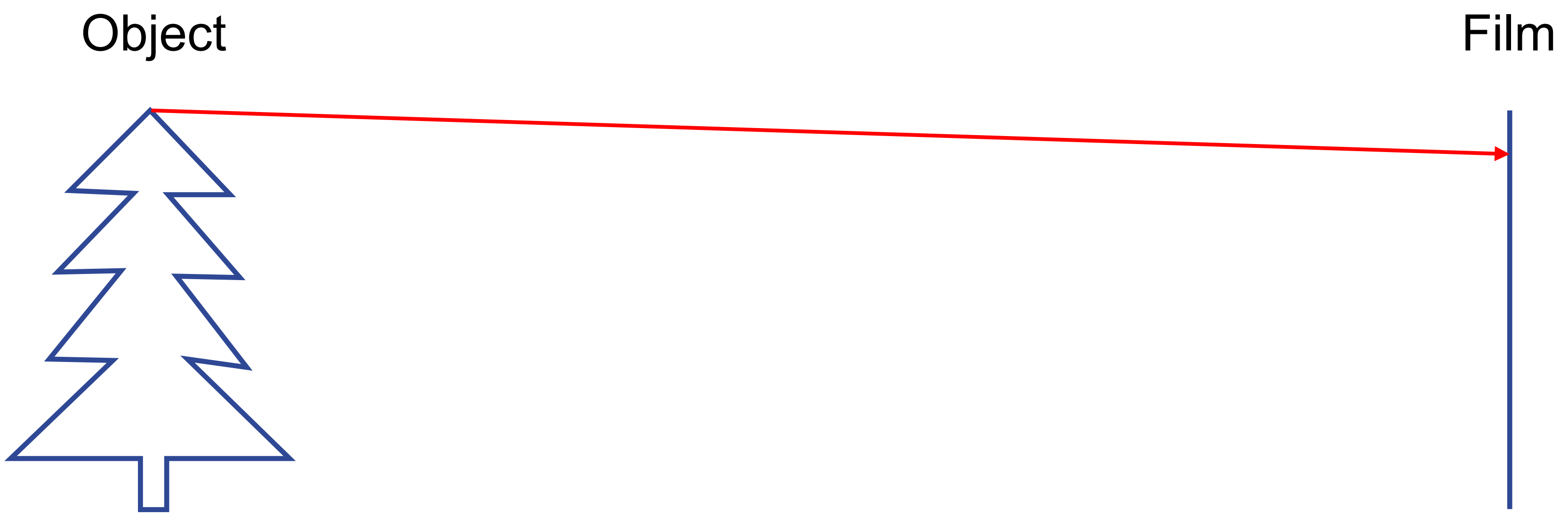

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Image formation

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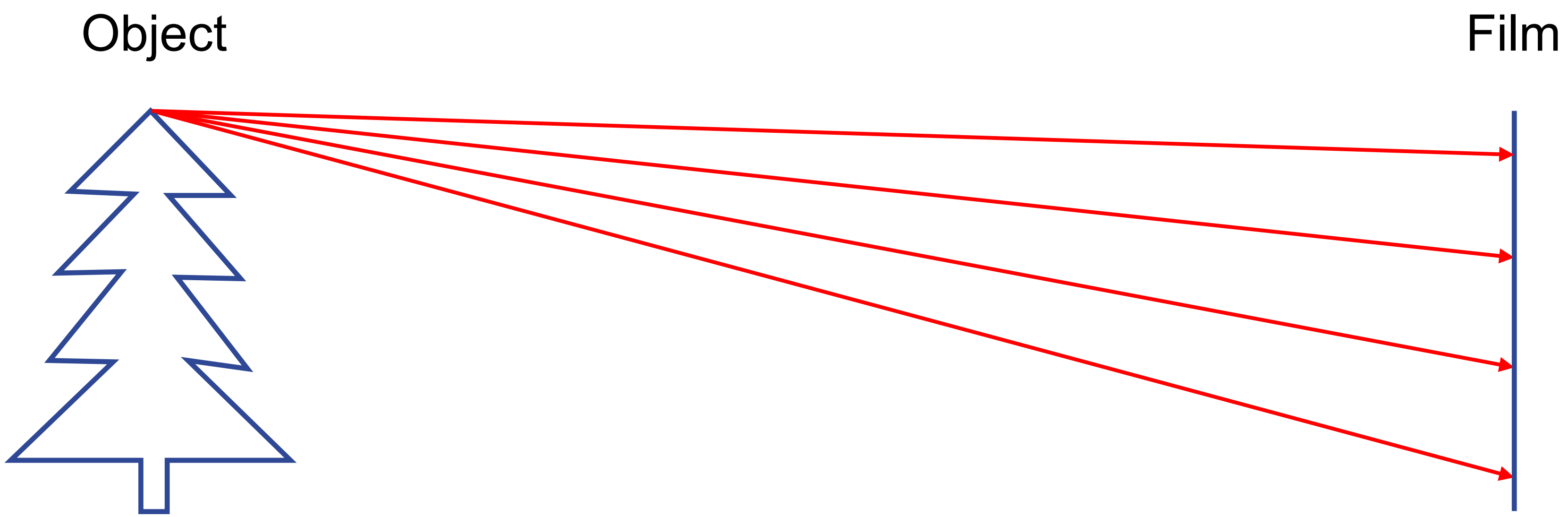
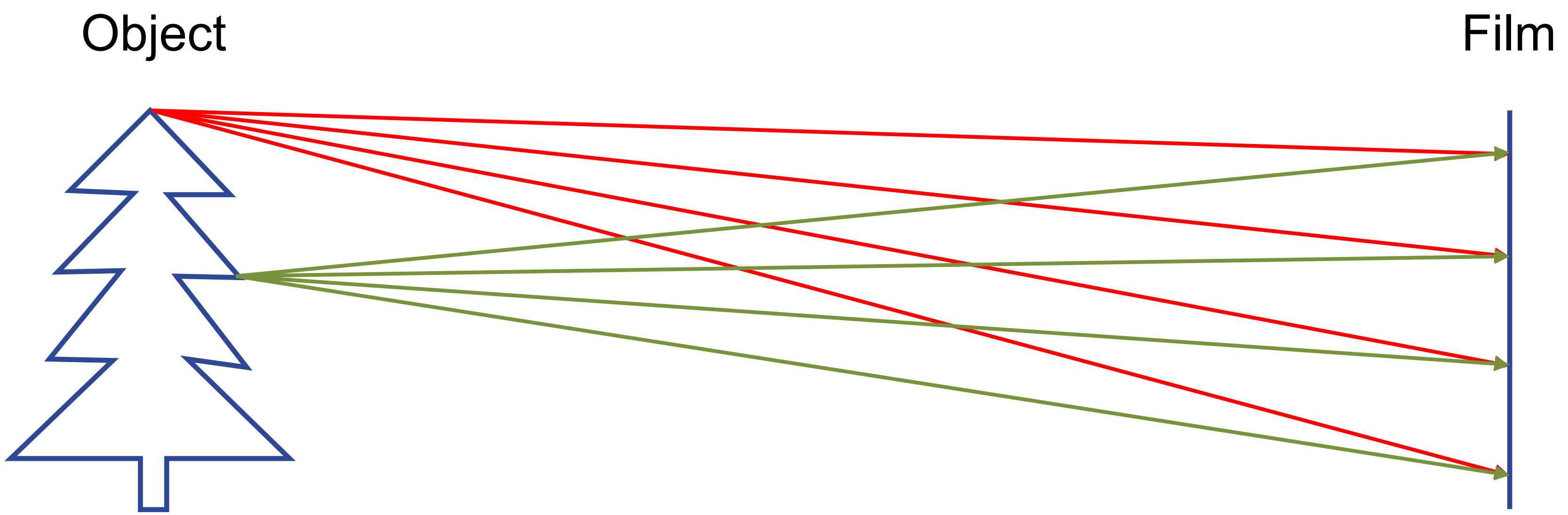


Do we get a reasonable image?

Image formation

Idea 1: Put a piece of film in front of an object

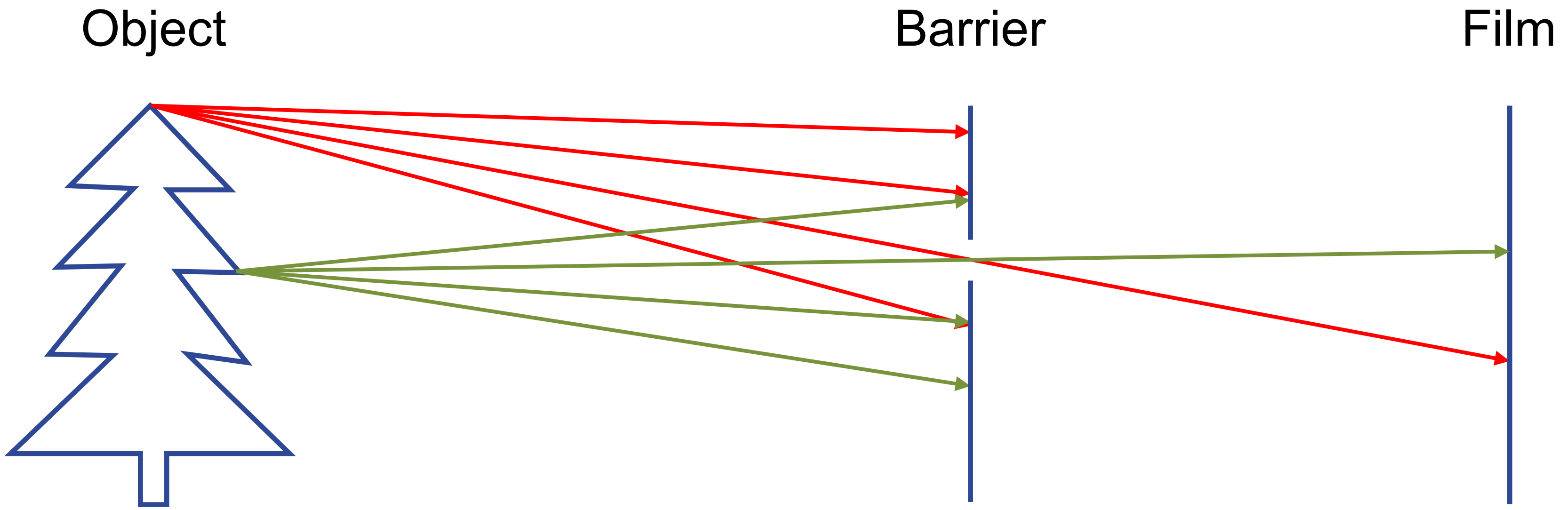


The resulting image is completely blurred

All object points are projected to all points on the film

Image formation

Idea 2: Add a barrier to block off most of the rays

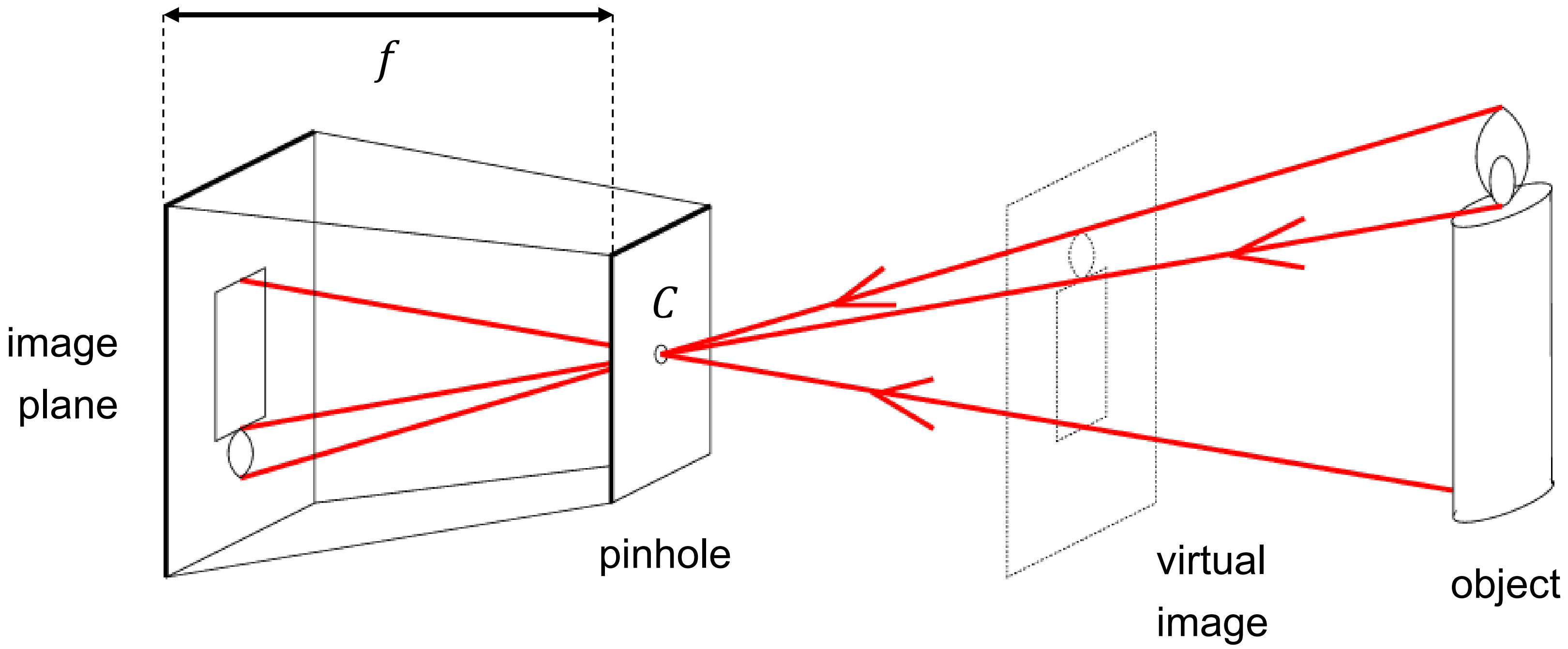


This reduces blurring

Opening known as the **pinhole** or **aperture**

Object points are projected to unique points on the film

Pinhole camera model



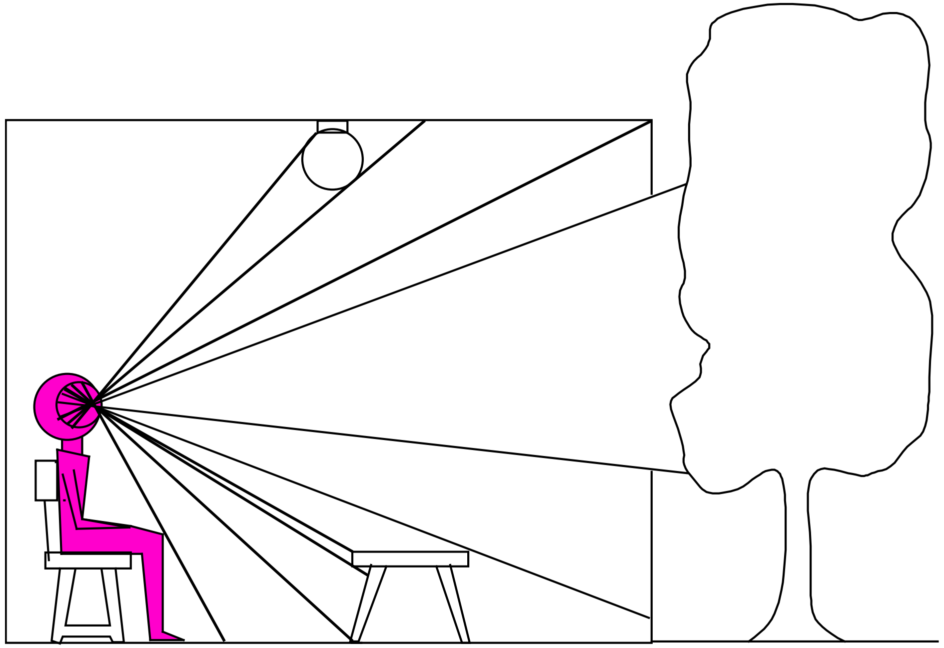
f = focal length

C = camera centre

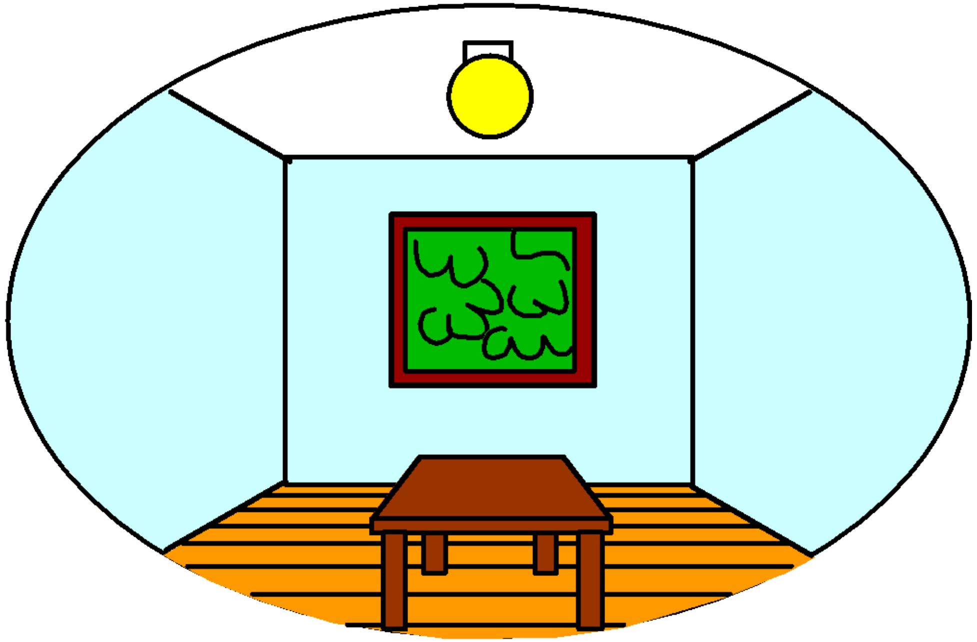
Figure from Forsyth

Dimensionality reduction machine

3D world



2D image



Projection can be tricky…

Projective geometry

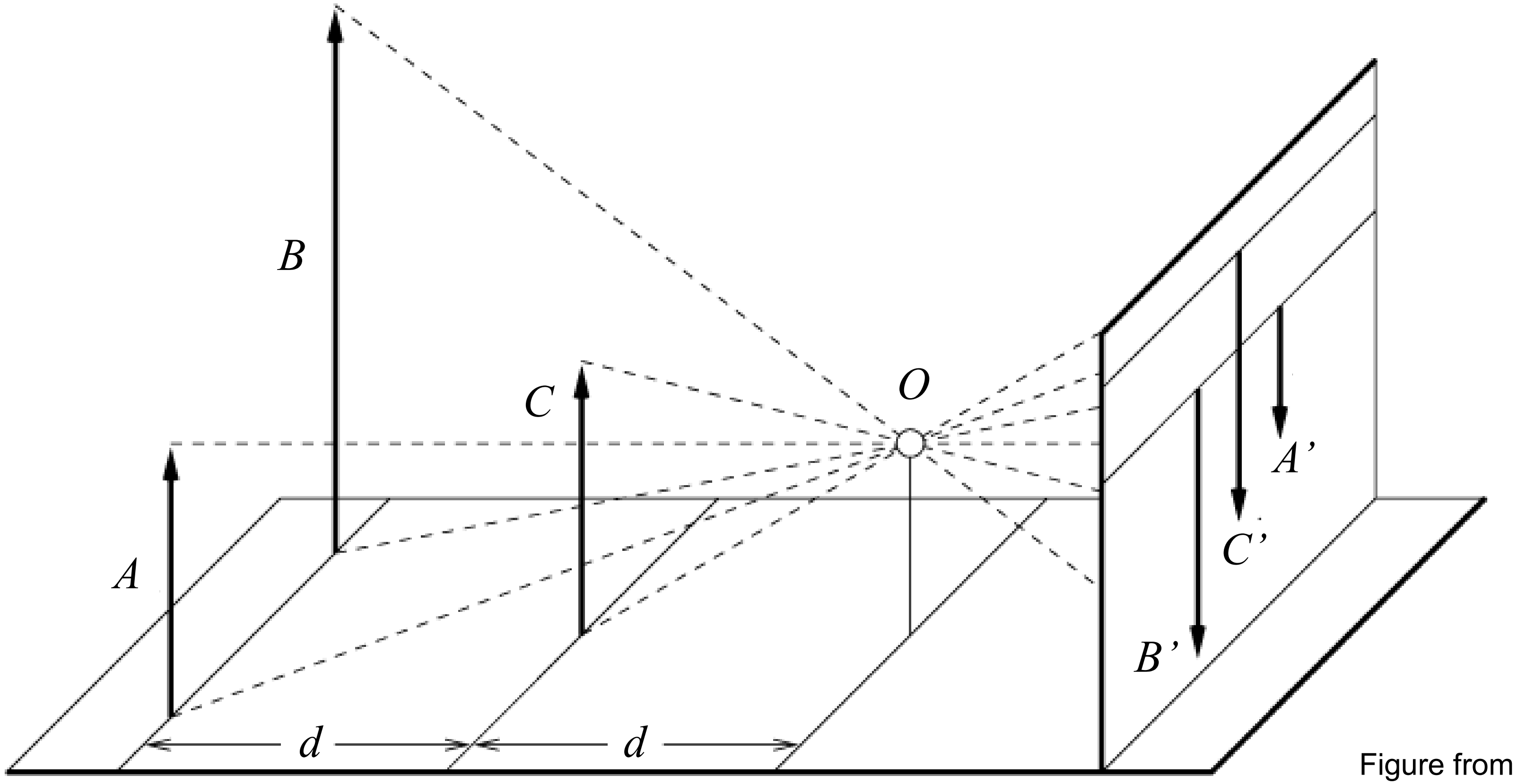
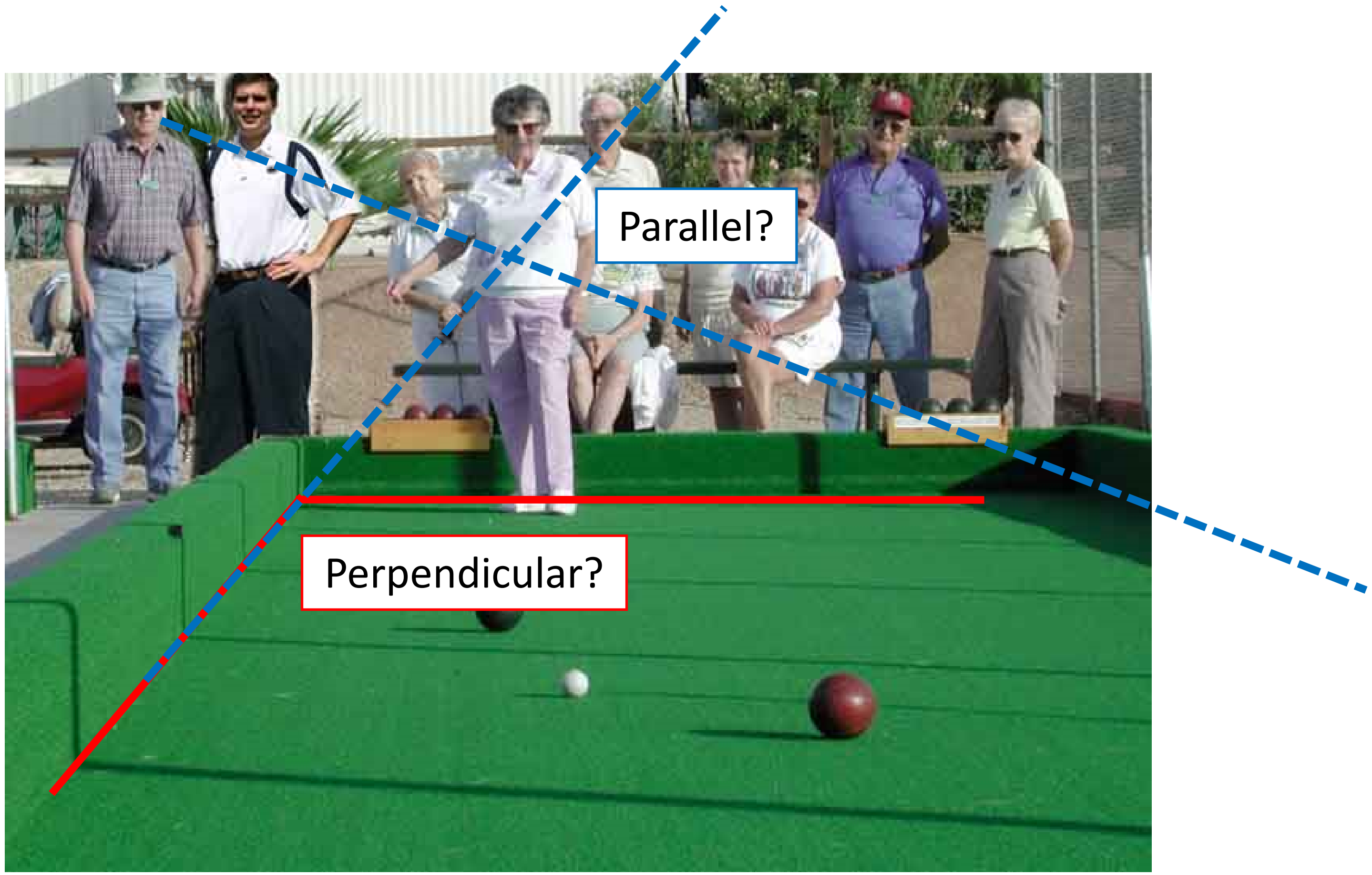


Figure from Forsyth

Length and area are not preserved

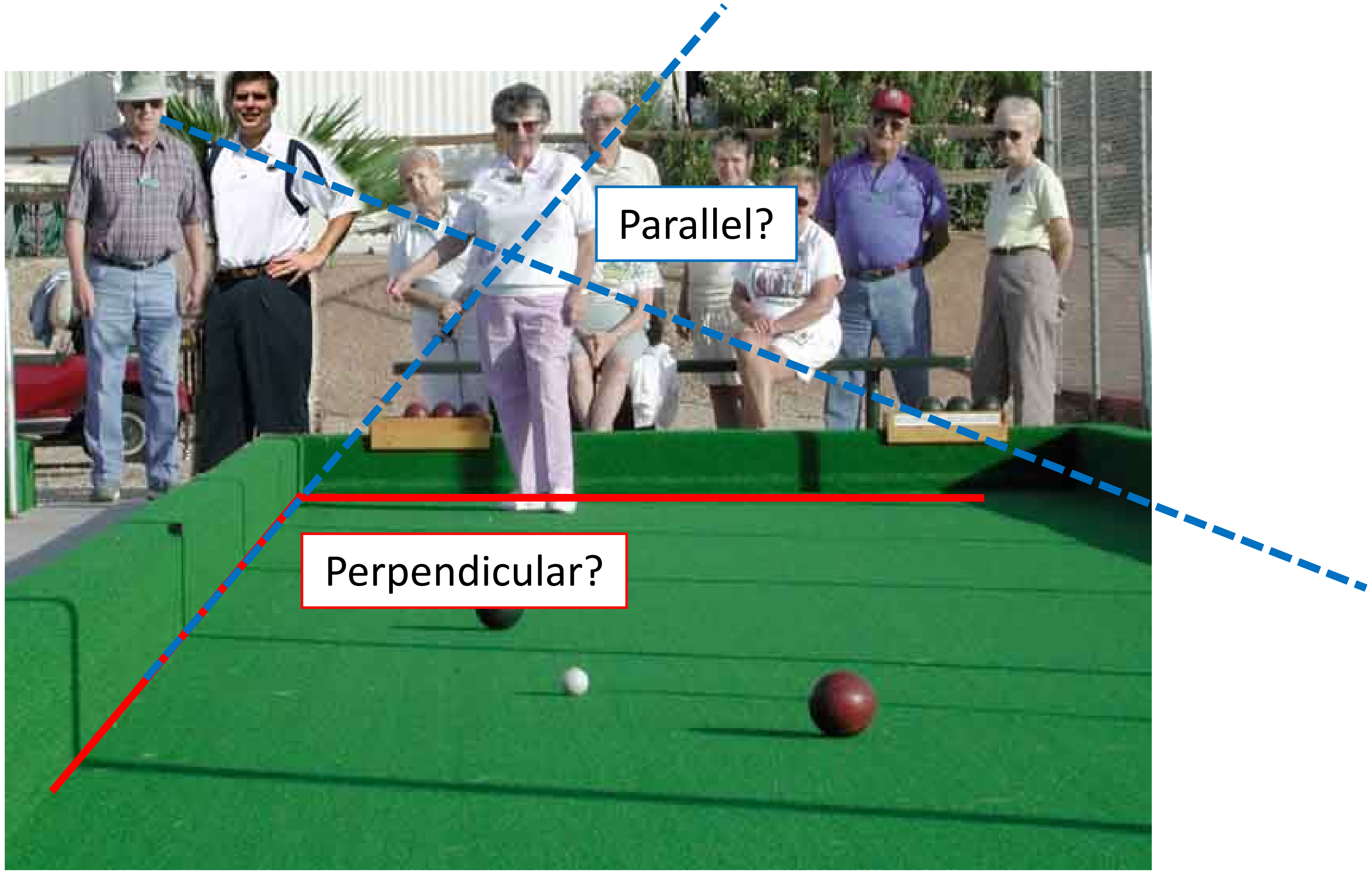
Projective geometry



What is lost?

Length and angles are not preserved

Projective geometry



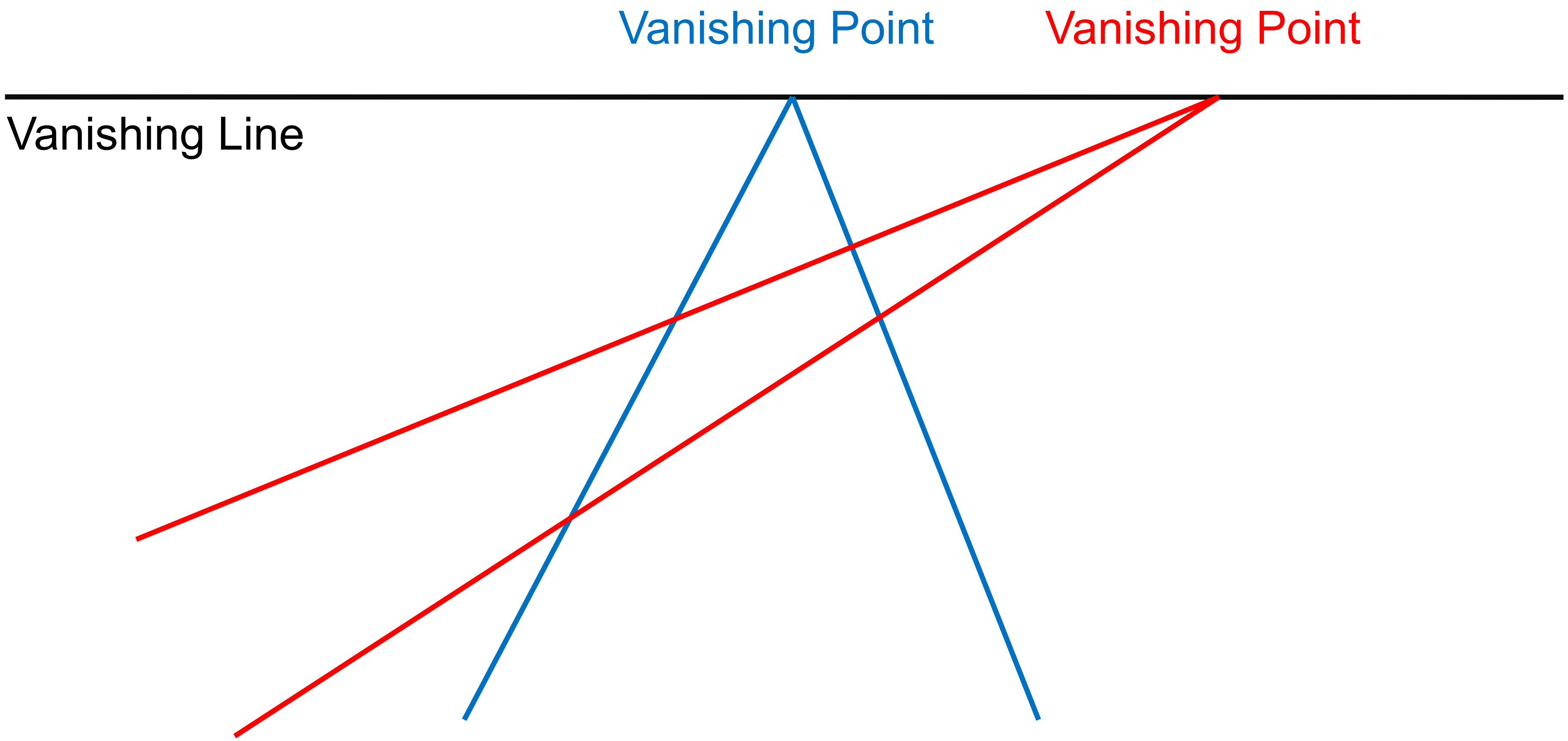
What is preserved?

Straight lines are still straight

Vanishing points and lines

Parallel lines in the 3D world
intersect in the 2D image
at a "vanishing point"

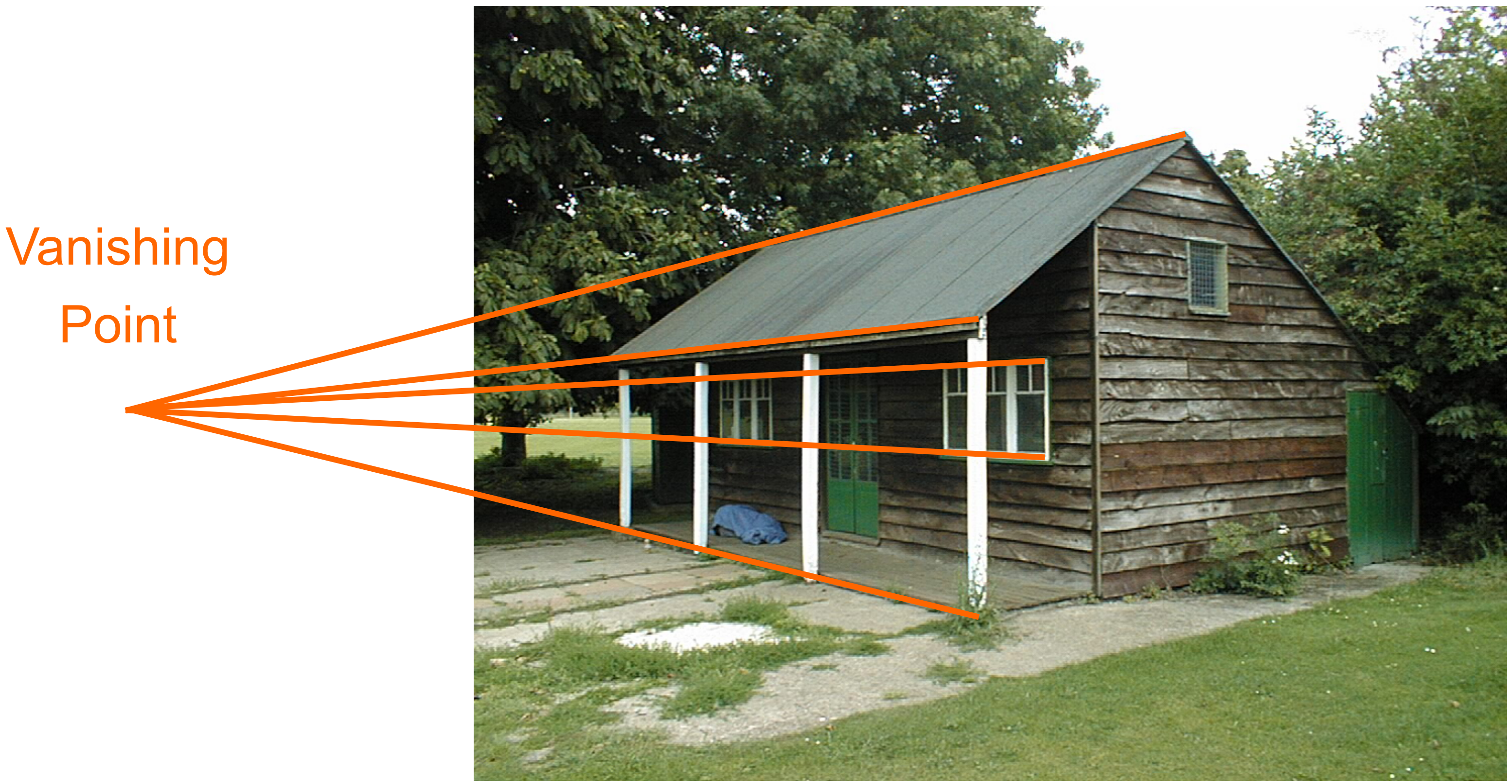
Vanishing points and lines



Vanishing points and lines

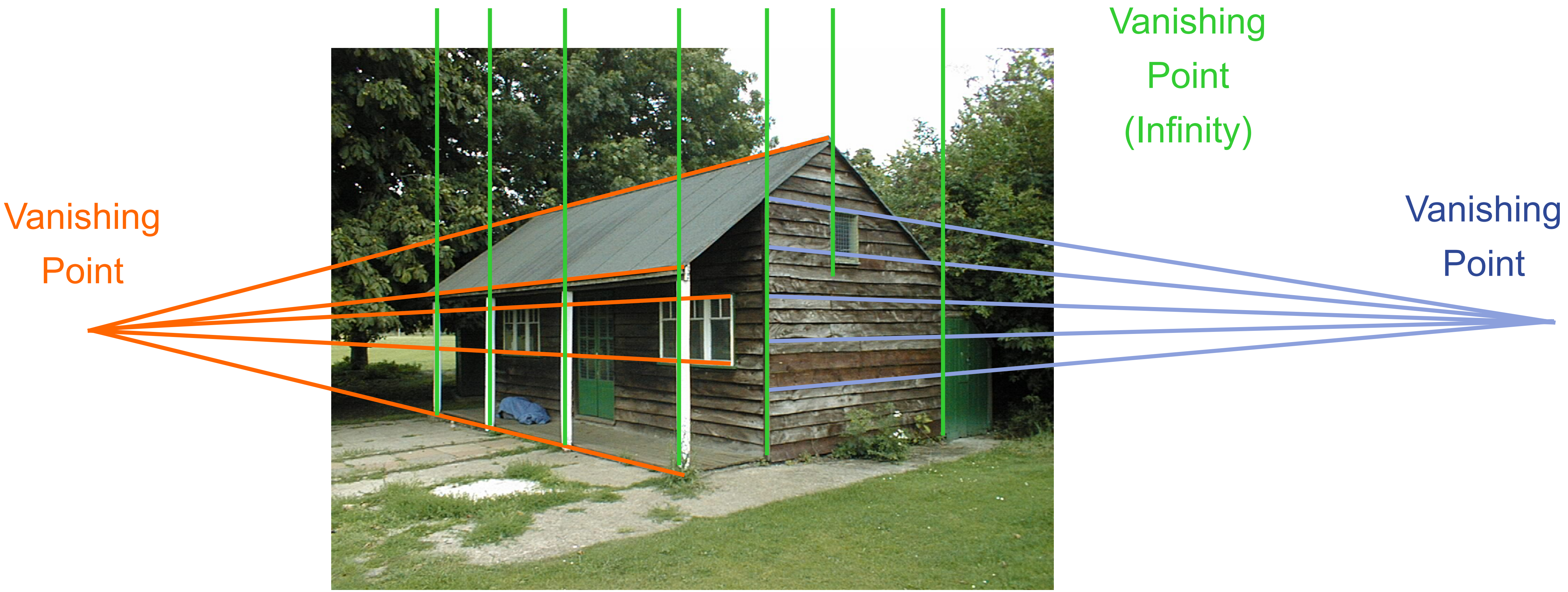
Photo from Criminisi

Vanishing points and lines



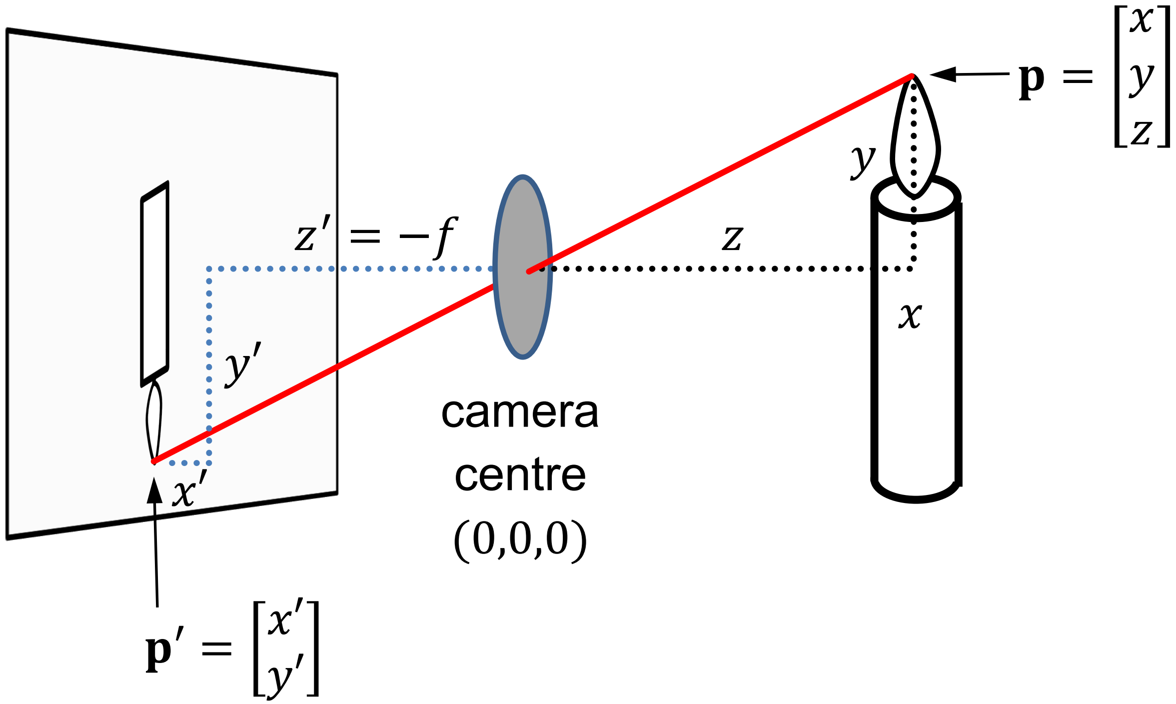
Vanishing points and lines

Vanishing points and lines



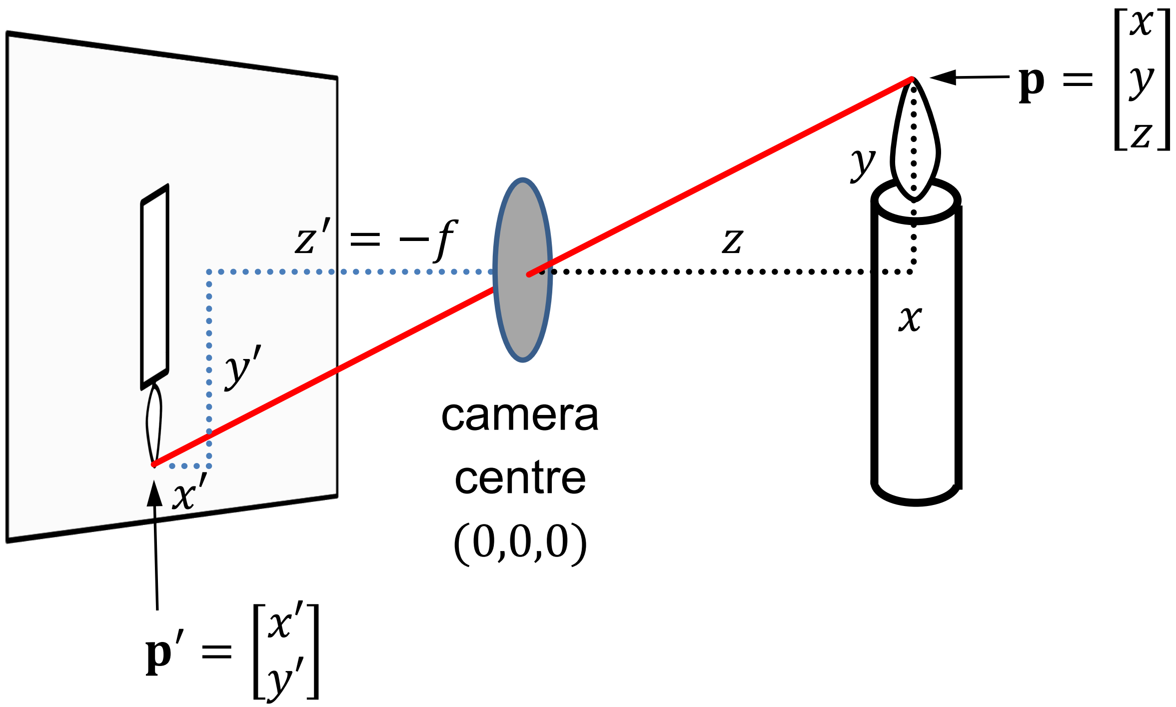
Projection mathematics

From **world coordinates** to **image coordinates**



Projection mathematics

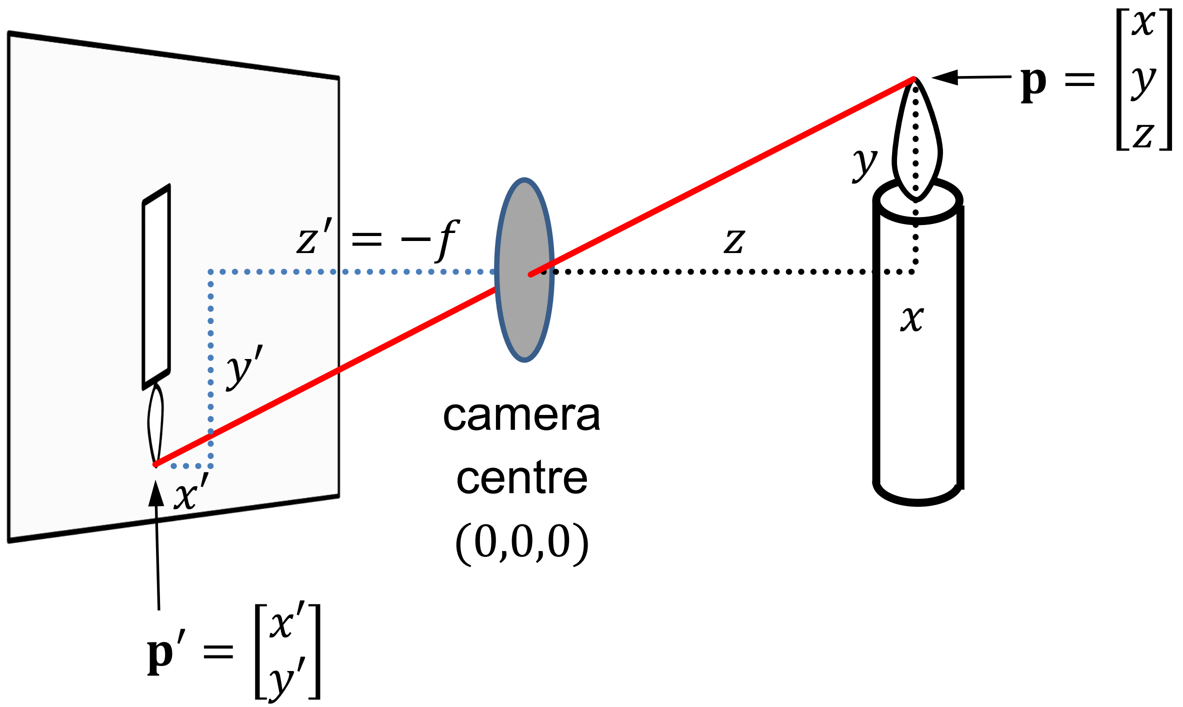
From **world coordinates** to **image coordinates**



If $x = 2, y = 3, z = 5$, and $f = 2$, what are x' and y' ?

Projection mathematics

From **world coordinates** to **image coordinates**



If $x = 2, y = 3, z = 5$, and $f = 2$, what are x' and y' ?

$$x' = -x \cdot \frac{f}{z} \qquad y' = -y \cdot \frac{f}{z}$$

$$\Rightarrow$$

$$x' = -2 \cdot \frac{2}{5} \qquad y' = -3 \cdot \frac{2}{5}$$

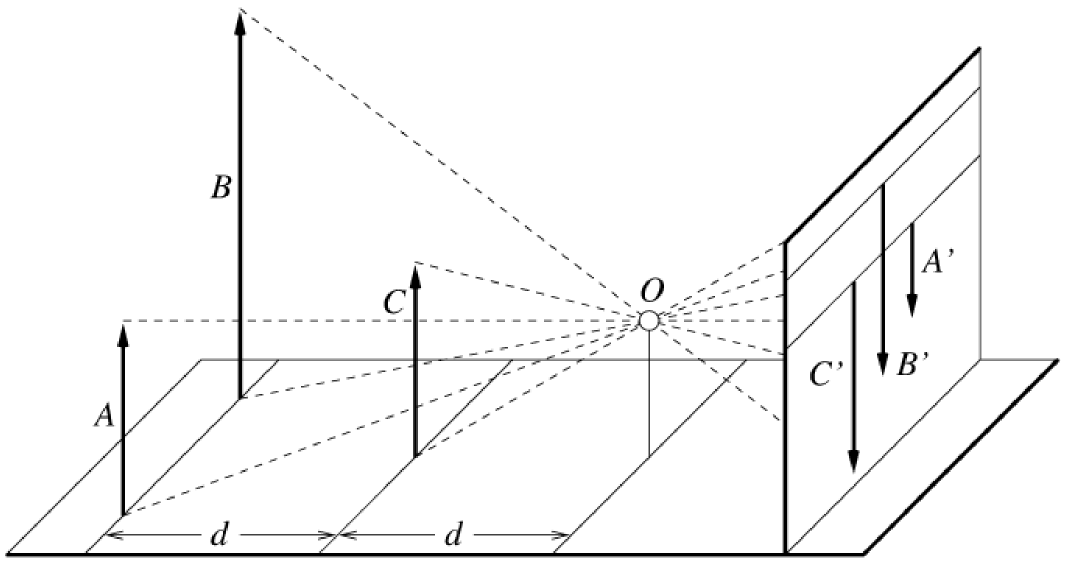
Perspective projection

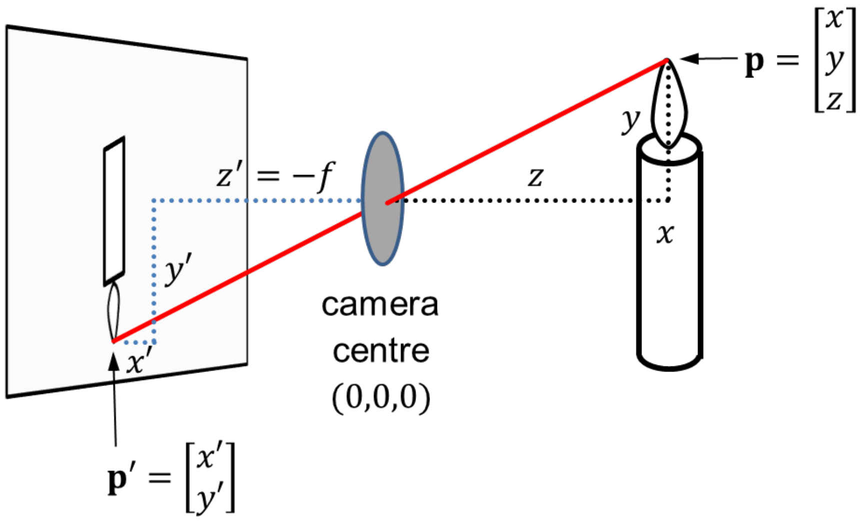
- Apparent size of object depends on its distance: far objects appear smaller
- By similar triangles:

  $$(x', y', z') = \left(-f\frac{x}{z}, -f\frac{y}{z}, -f\right)$$

- Ignore third coordinate and mirror:

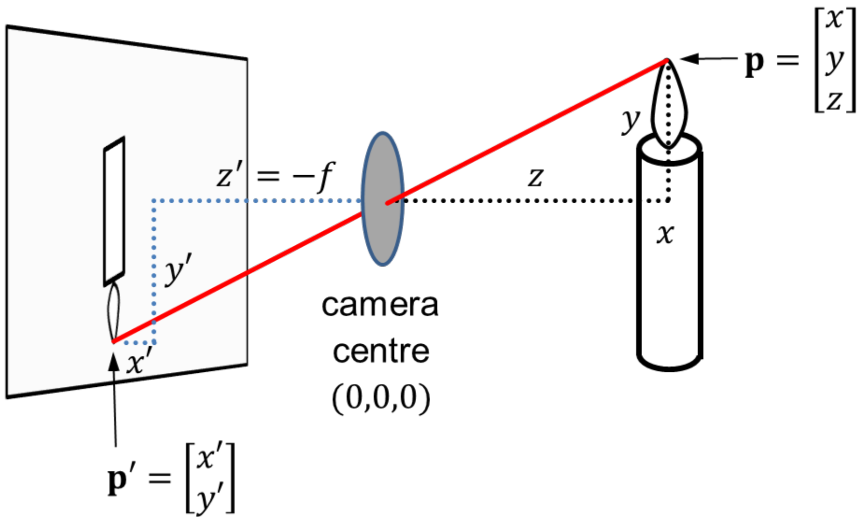
  $$(x', y') = \left(f\frac{x}{z}, f\frac{y}{z}\right)$$





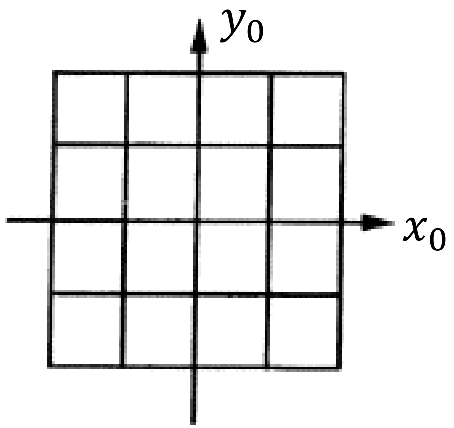
Affine projection

- Suitable when scene depth is small relative to average distance from camera
- Let magnification $m = f/z_0$ be a positive constant and all points in the scene have approximately constant distance z_0 to the camera
- Leads to weak perspective projection:
  $(x', y') = (mx, my)$
- Orthographic projection when $m = 1$:
  $(x', y') = (x, y)$

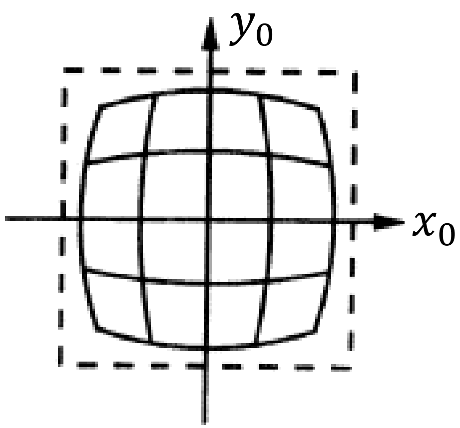


Beyond pinholes: radial distortions

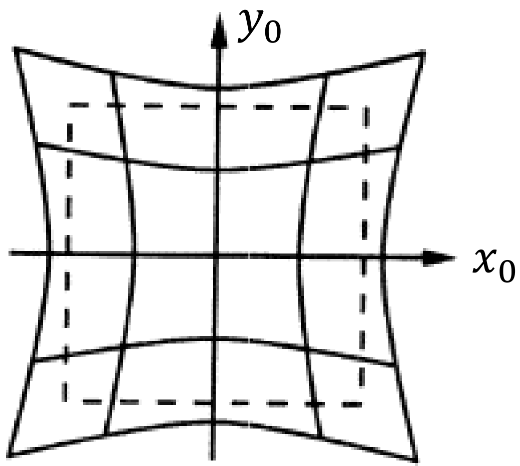
- Modern cameras use lenses instead of pinholes



No distortion



Barrel distortion



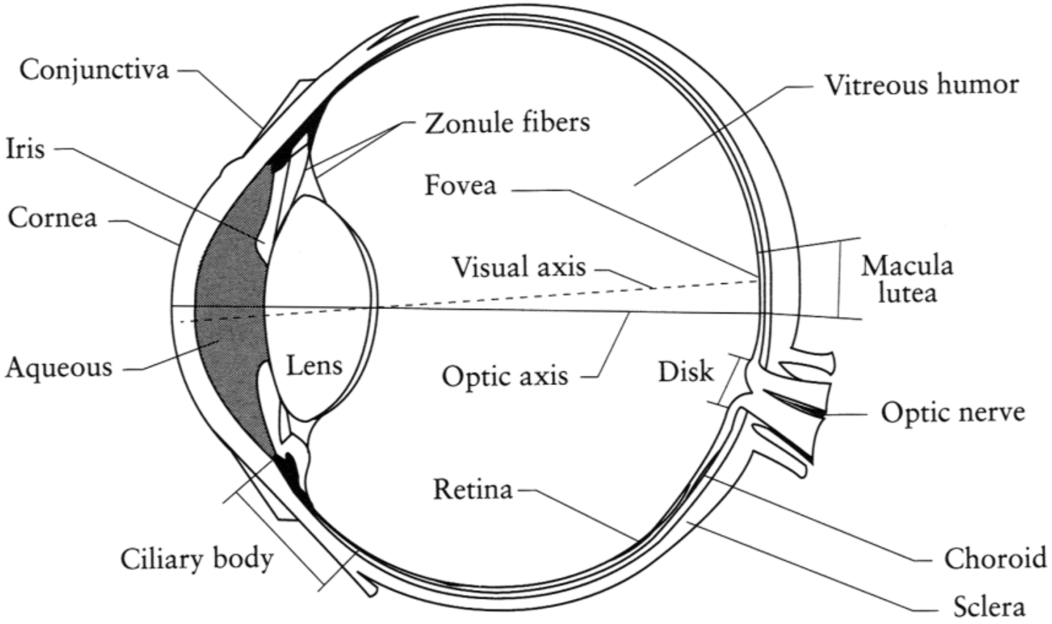
Pincushion distortion

Image from Martin Habbecke

Barrel distortion corrected

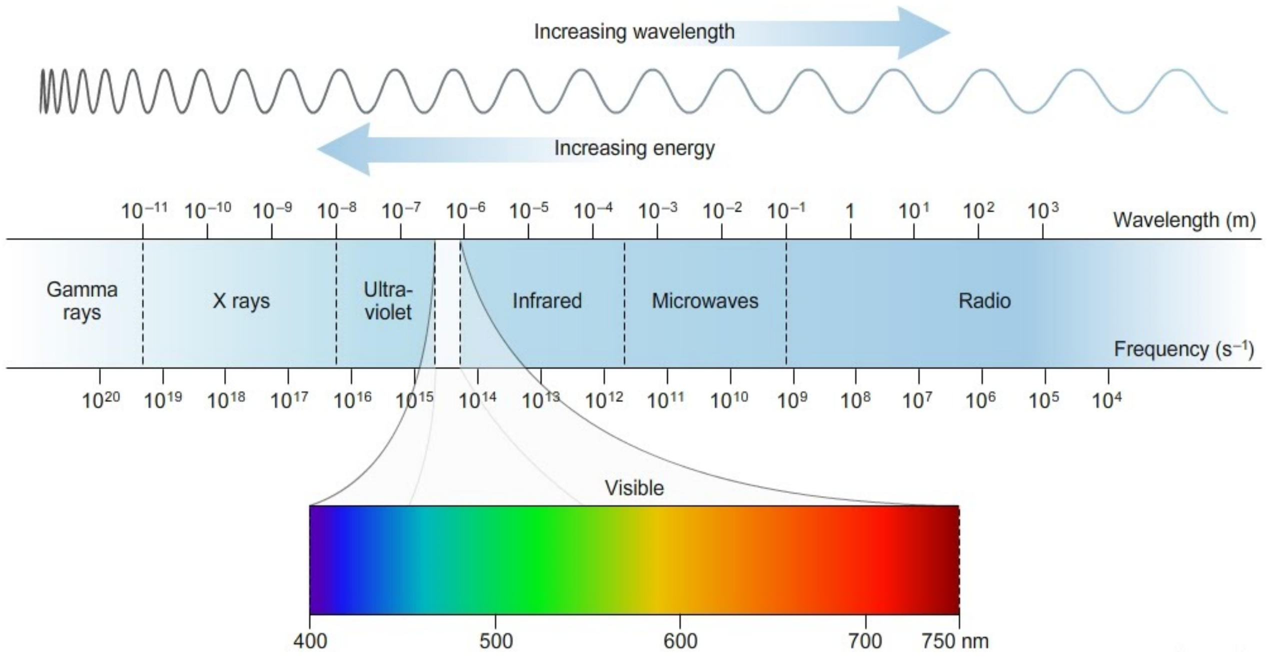
Comparing with human vision

- Cameras imitate the frequency response of the human eye so it is good to know something about it
- Computer vision probably would not get as much attention if biological vision (especially human vision) had not proven that it is possible to make important judgements from 2D images

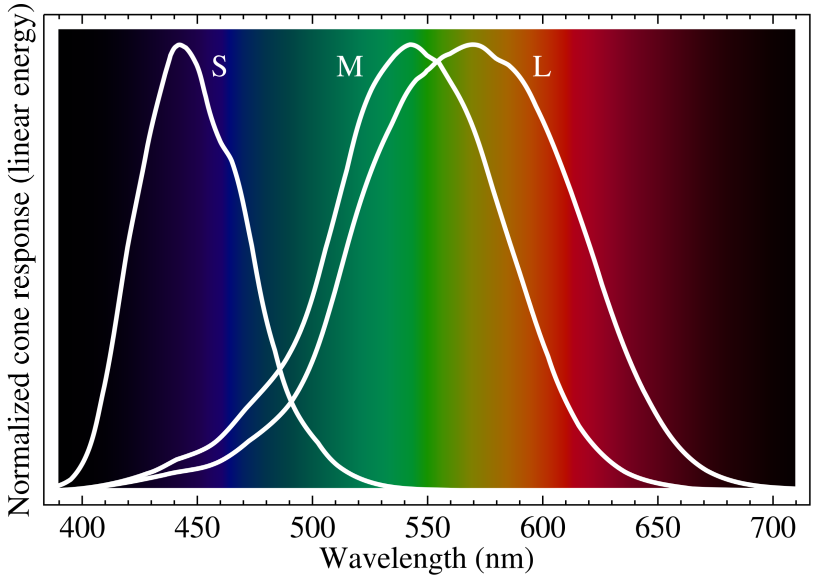


The Eye

Electromagnetic spectrum



https://sites.google.com/site/chempendix/em-spectrum

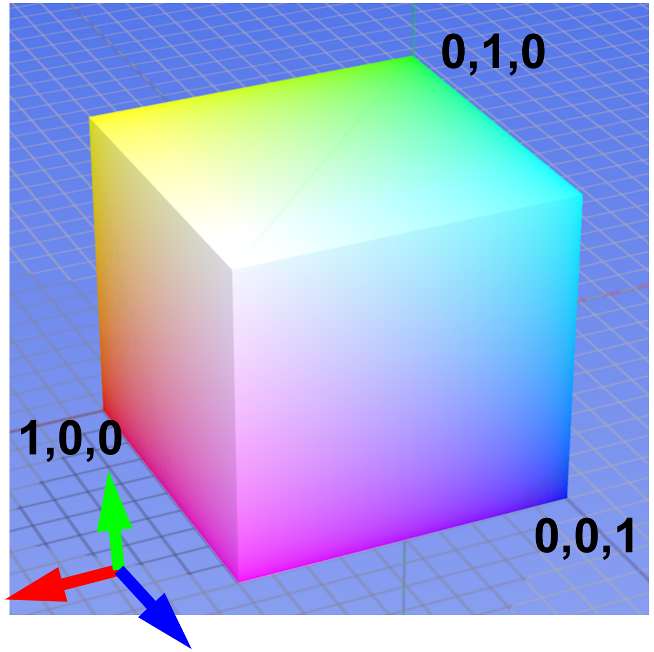


Normalized responsivity spectra of human cone cells (S, M, L types)

Colour represented by RGB images

Colour spaces: RGB

Default colour space in vision



[Source: Wikipedia]

Drawback: strongly correlated channels

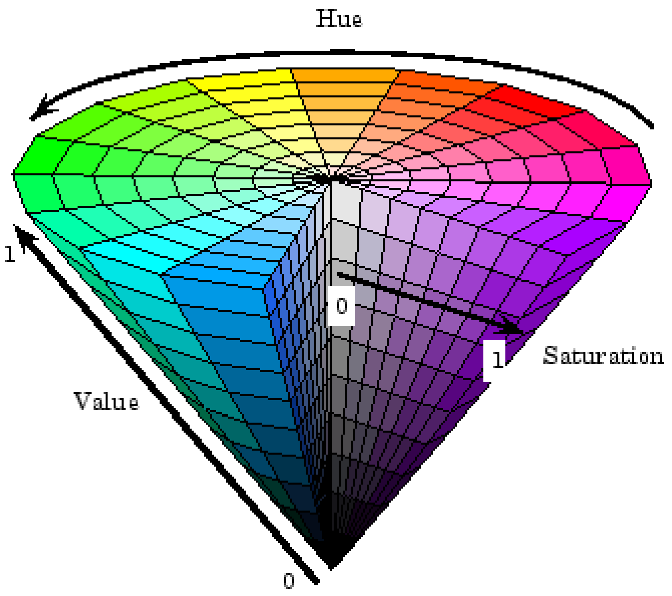
R
(G=0,B=0)

G
(R=0,B=0)

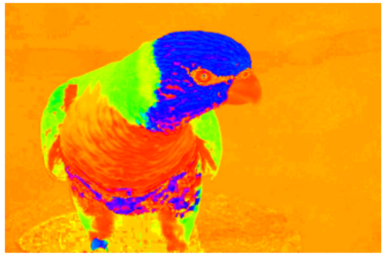
B
(R=0,G=0)

Colour spaces: HSV

Intuitive colour space



Drawback: confounded channels



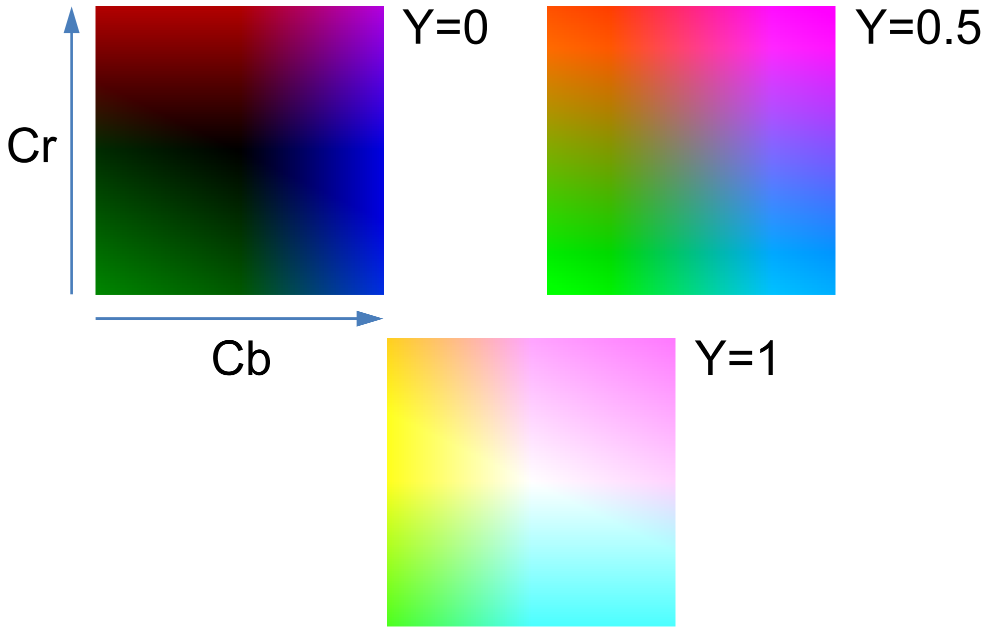
H
(S=1,V=1)

S
(H=1,V=1)

V
(H=1,S=0)

Colour spaces: YCbCr

Fast to compute, good for compression, used by TV



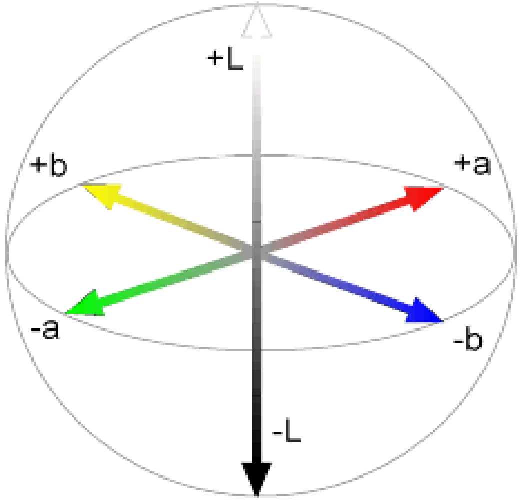
Y
(Cb=0.5,Cr=0.5)

Cb
(Y=0.5,Cr=0.5)

Cr
(Y=0.5,Cb=0.5)

Colour spaces: L*a*b*

"Perceptually uniform" colour space



Any numerical change corresponds to similar perceived change in color: Euclidean distances make sense

a.k.a. CIELAB

L

(a=0, b=0)

a

(L=65, b=0)

b

(L=65, a=0)

Digital image formation

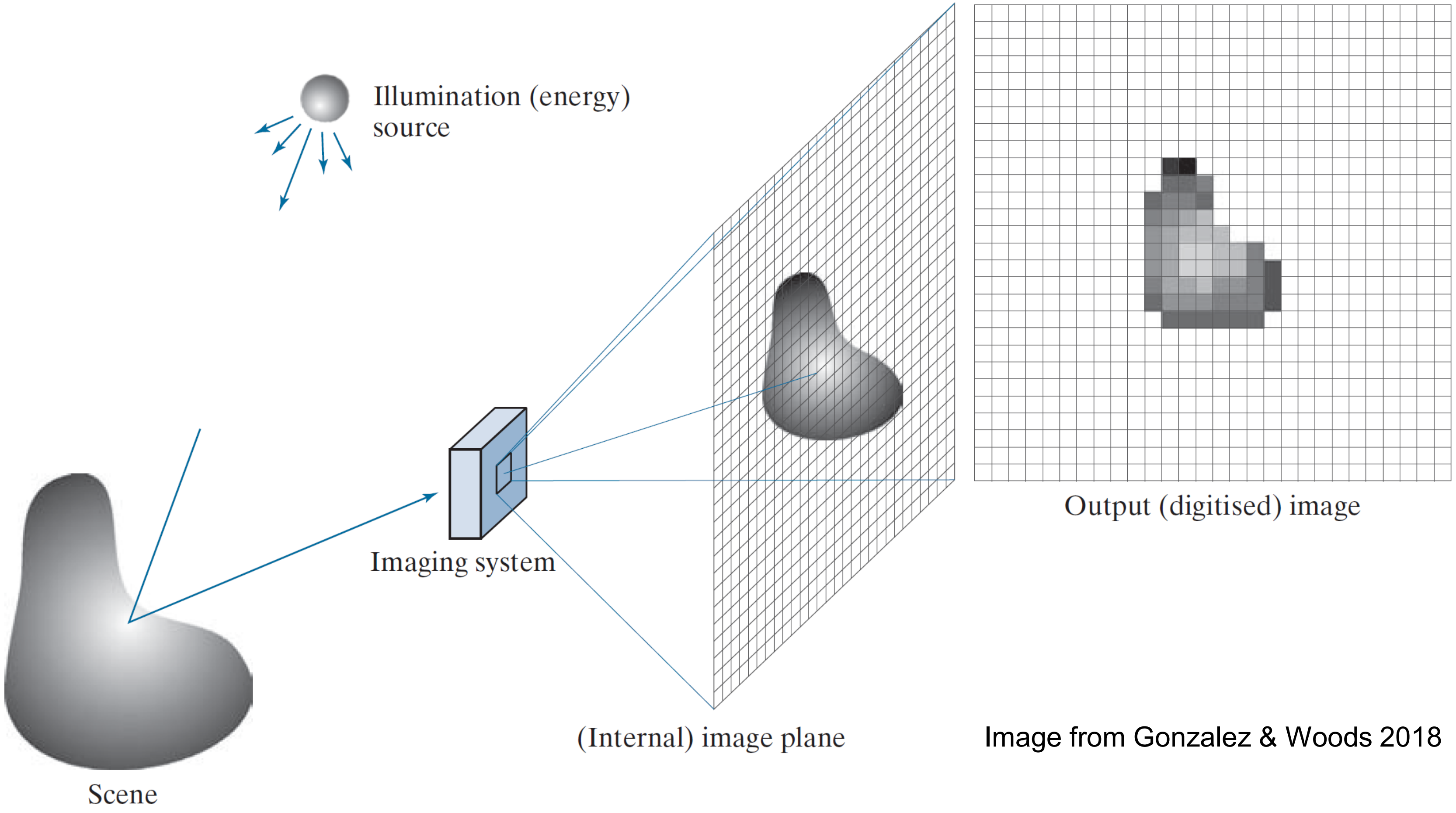
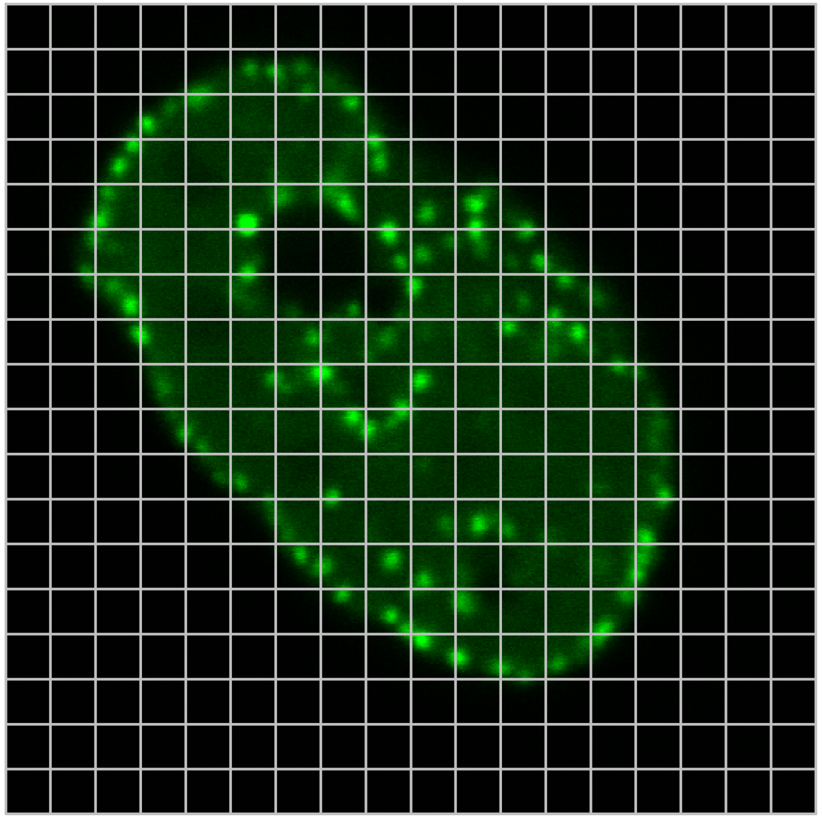


Image from Gonzalez & Woods 2018

Digital image formation



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0	2	25	172	181	133	133	164	90	14	5	2	2	0	0	0	0	0
2	5	175	130	104	127	141	164	206	65	31	11	2	2	0	0	0	0
2	28	212	124	110	204	164	232	133	155	218	87	14	2	2	0	0	0
2	73	178	133	121	195	34	31	198	175	204	167	104	14	5	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Displaying a digital image

0	2	2	2	5	8	11	8	2	2	0	0	0	0	0	0	0	0
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0	2	25	172	181	133	133	164	90	14	5	2	2	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
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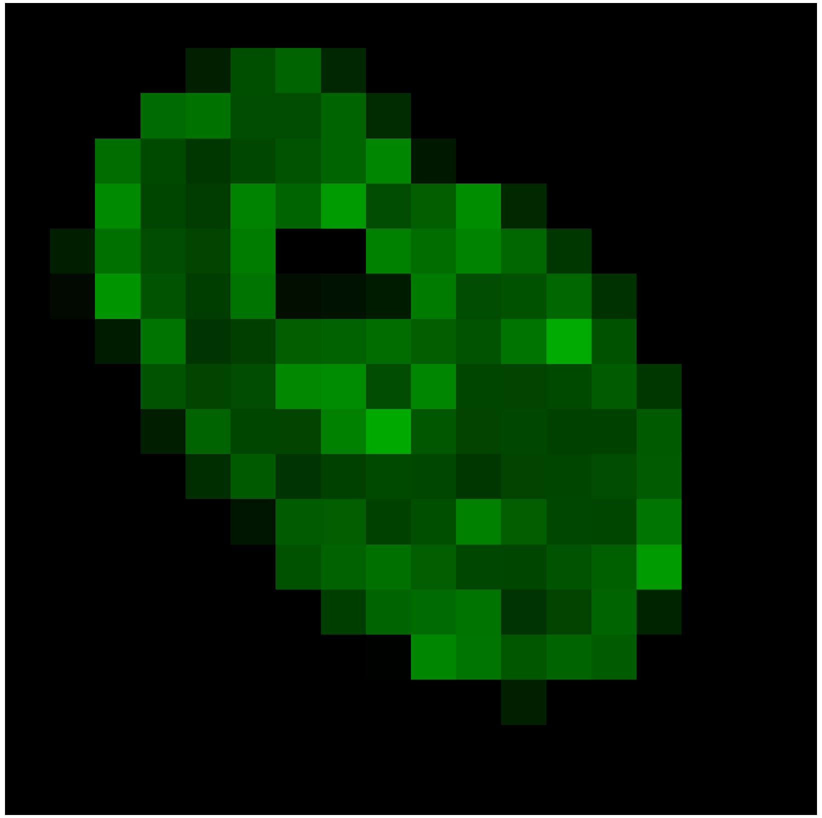
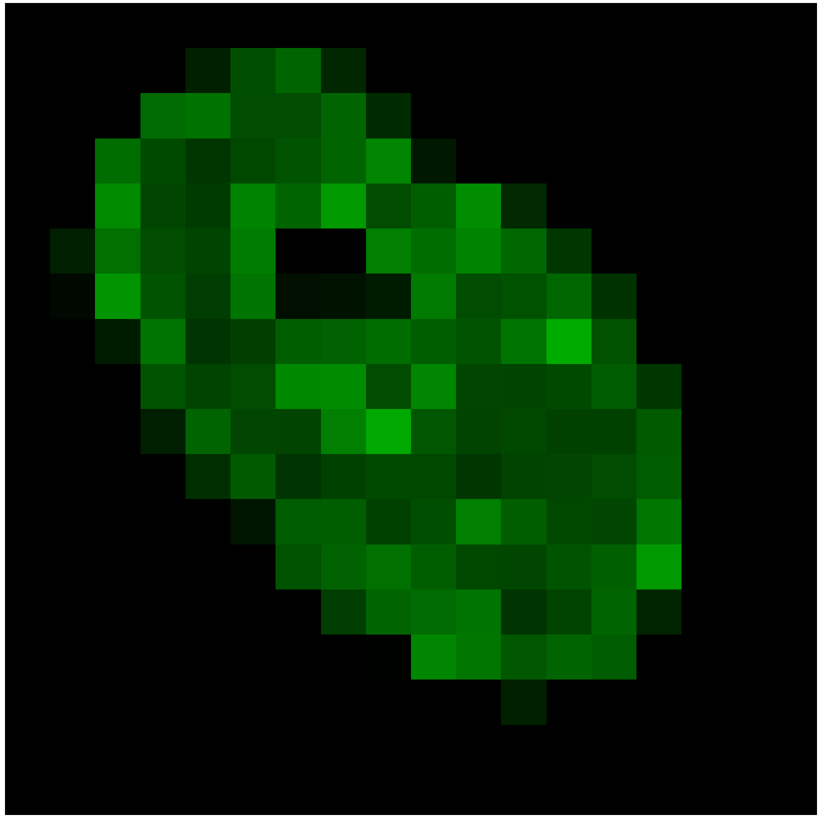


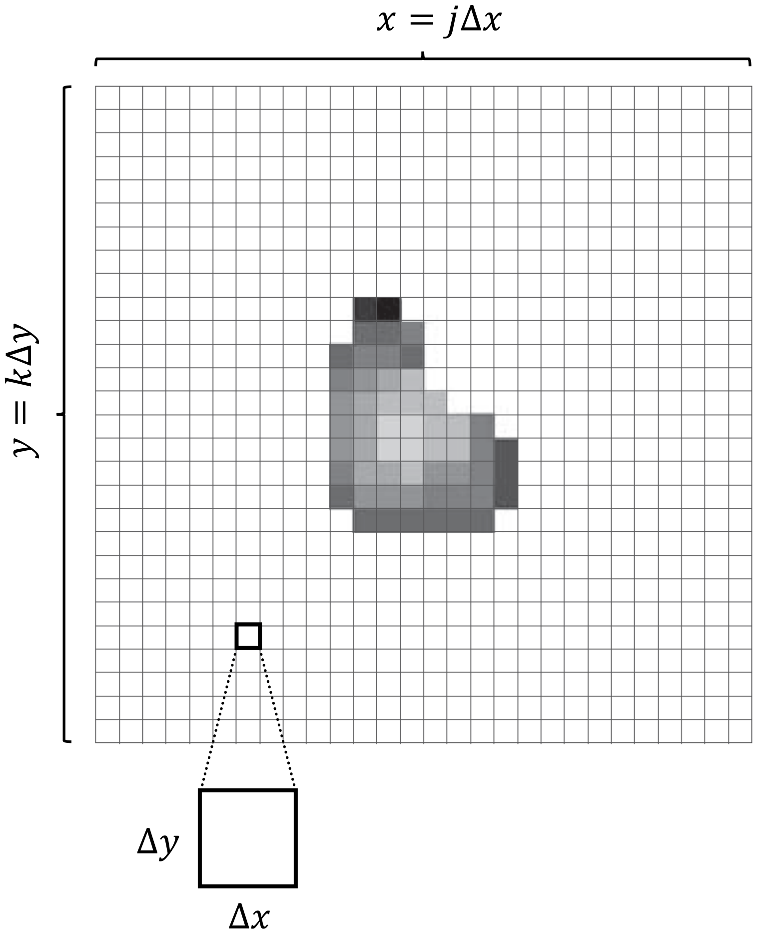
Comparing the original and digital image



Digitisation by spatial sampling

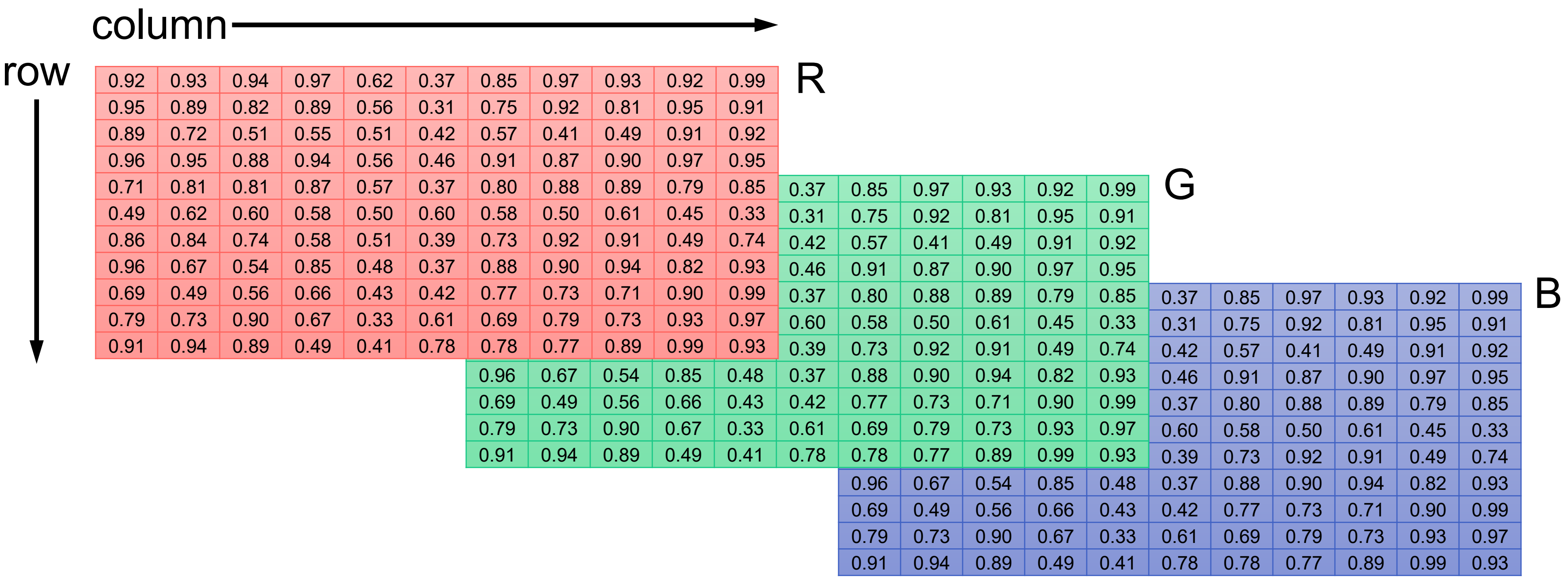
- Digitisation converts an analog image to a digital image by sampling the image space
- Sampling discretises the coordinates x and y
  - Spatial discretisation of a picture function $f(x, y)$
  - Typically a rectangular grid of sampling points is used

    $x = j\Delta x,\ y = k\Delta y$ for $j = 0 \dots M-1,\ k = 0 \dots N-1$
  - The Δx and Δy are called the sampling intervals



Digital colour images

Each channel is a digital image with the same number of rows and columns



Spatial resolution

- Spatial resolution: number of pixels per unit of length
- Example: resolution decreases by one half each time (see right)
- Human faces can be recognized at 64 x 64 pixels per face
- Appropriate resolution is essential:
  - Too little resolution yields poor recognition
  - Too much resolution is slow and wastes memory


1/1


1/2


1/4

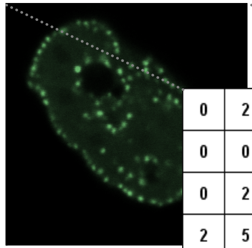

1/8


1/16

Quantisation

- Quantisation digitises the image intensity or amplitude values $f(x, y)$
  - Called intensity or gray-level quantisation
  - Gray-level resolution to be chosen per application
    - ❖ For example, 16, 32, 64, ..., 128, 256 levels
    - ❖ Should be high enough for human perception of shading details
    - ❖ The latter requires about 100 levels for a realistic image
    - ❖ Should not be higher than necessary to avoid wasting storage

Quantisation and bits per pixel



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0	2	25	172	181	133	133	164	90	14	5	2	2	0	0	0	0	0
2	5	175	130	104	127	141	164	206	65	31	11	2	2	0	0	0	0
2	28	212	124	110	204	164	232	133	155	218	87	14	2	2	0	0	0
2	73	178	133	121	195	34	31	198	175	204	167	104	14	5	0	0	0
2	45	226	141	113	184	53	59	70	192	133	138	167	99	11	2	0	0
0	2	70	184	102	116	155	161	175	155	141	184	255	138	34	5	2	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

→ **Pixel** (picture element)

Levels per pixel:

8 bits = 2^8 = 256

12 bits = 2^{12} = 4,096

16 bits = 2^{16} = 65,536

24 bits = 2^{24} = 16,777,216

Further reading on discussed topics

- Chapter 2 of Szeliski
- Chapter 2 of Shapiro and Stockman

Acknowledgements

- Several slides from Derek Hoiem, Alexei Efros, Steve Seitz, and David Forsyth
- Some material drawn from referenced and associated online sources
- Image sources credited where possible