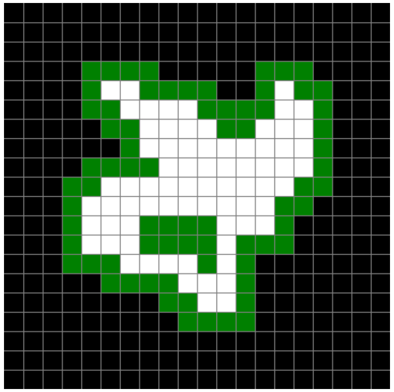
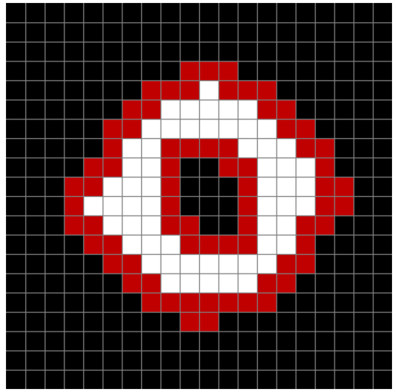
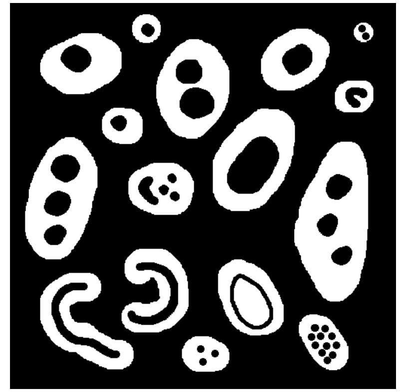
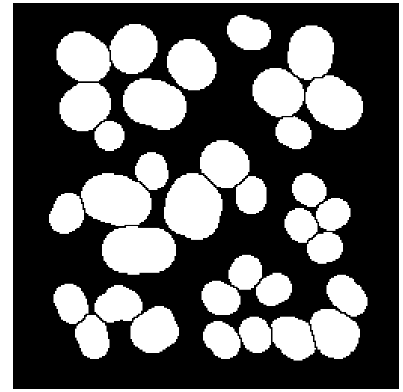





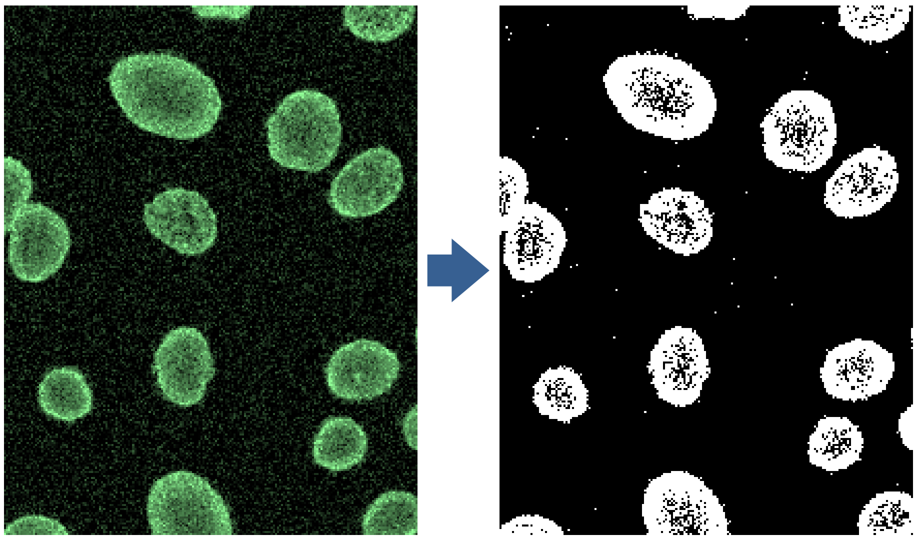
COMP9517: Computer Vision

Image Segmentation

Part 2

How to improve image segmentation results?

Processing using mathematical morphology



How to clean up background noise?

How to clean up object noise?

How to separate touching objects?

How to close holes in objects?

How to extract object contours?

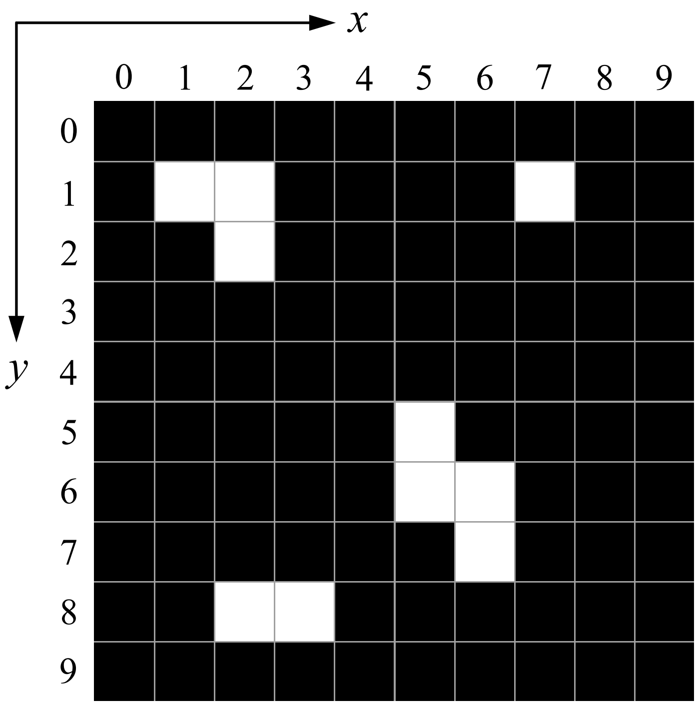
How to compute distances?

…

- Binary mathematical morphology (applies to binary images)
- Gray-scale mathematical morphology (applies to gray-scale images)

Nonlinear image processing based on set-theory rather than on calculus

Binary image representations



$$\begin{bmatrix} 0&0&0&0&0&0&0&0&0&0\\ 0&1&1&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&1&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&1&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$I = \{(1,1),(2,1),(7,1),(2,2),(5,5),(5,6),(6,6),(6,7),(2,8),(3,8)\}$$

Binary image	**Image as matrix**	**Image as set**
white = foreground	1 = foreground	
black = background	0 = background	

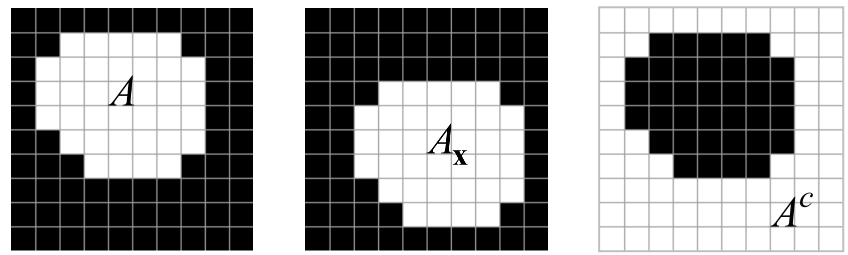
Basic set operations

Given sets $A = \{\mathbf{a}_i\}_{i=1}^{N_A}$ **and** $B = \{\mathbf{b}_i\}_{i=1}^{N_B}$**, with** $\mathbf{a}_i, \mathbf{b}_i \in \mathbf{Z}^n$**, we have**

Translation: $A_{\mathbf{x}} = \{\mathbf{c}_i \mid \mathbf{c}_i = \mathbf{a}_i + \mathbf{x},\ \mathbf{a}_i \in A\}$ for any given $\mathbf{x} \in \mathbf{Z}^n$

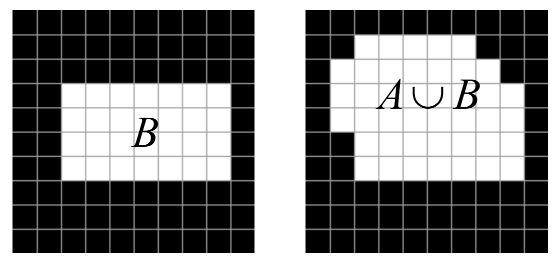
Reflection: $A^r = \{\mathbf{x}_i \mid \mathbf{x}_i = -\mathbf{a}_i,\ \mathbf{a}_i \in A\}$

Complement: $A^c = \{\mathbf{x}_i \mid \mathbf{x}_i \notin A\}$



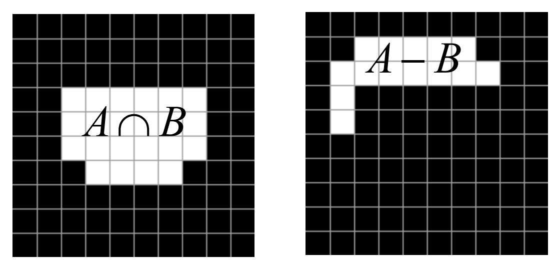
Union: $A \cup B = \{\mathbf{x}_i \mid \mathbf{x}_i \in A \vee \mathbf{x}_i \in B\}$

Intersection: $A \cap B = \{\mathbf{x}_i \mid \mathbf{x}_i \in A \wedge \mathbf{x}_i \in B\}$



Difference: $A - B = \{\mathbf{x}_i \mid \mathbf{x}_i \in A \wedge \mathbf{x}_i \notin B\}$

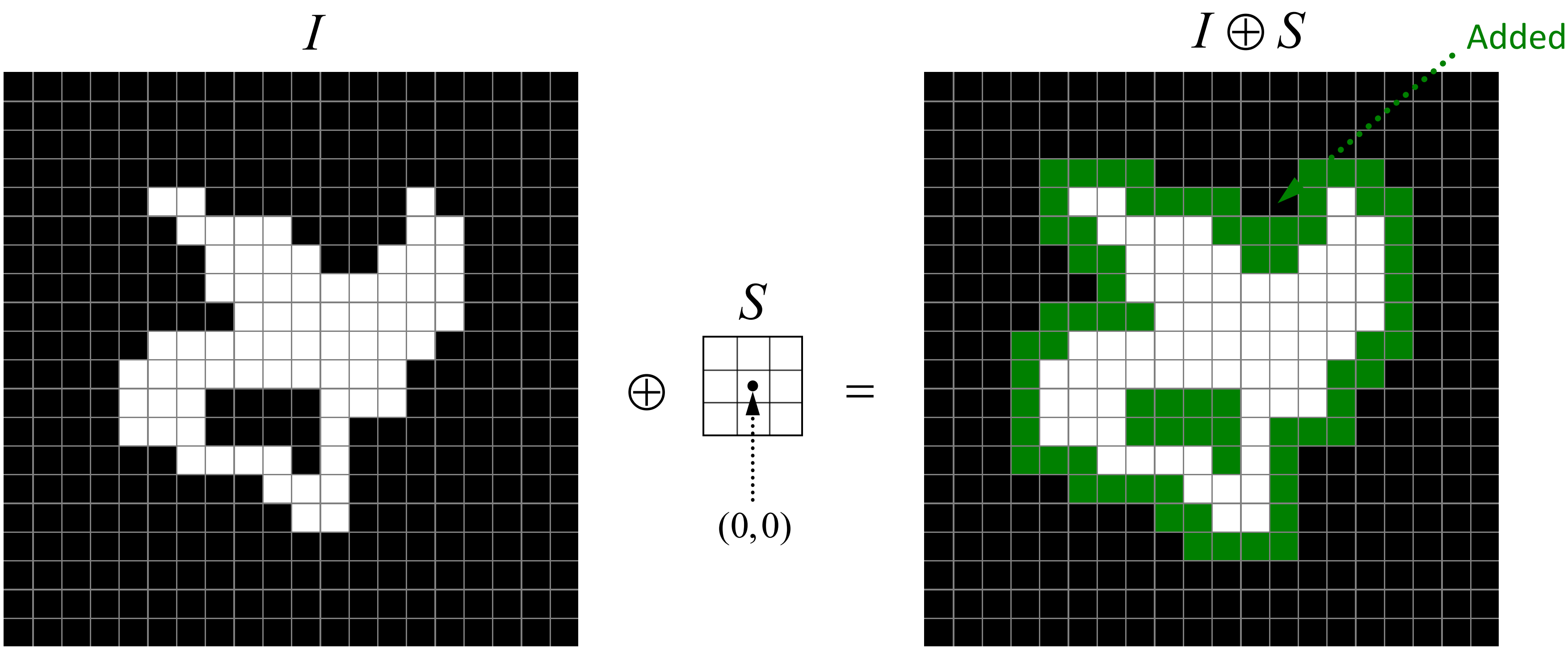
Cardinality: $|A| = N_A$ and $|B| = N_B$



Dilation of binary images

Definition of binary dilation: $I \oplus S = \left\{ \mathbf{x} \mid S_{\mathbf{x}}^{r} \cap I \neq \varnothing \right\}$

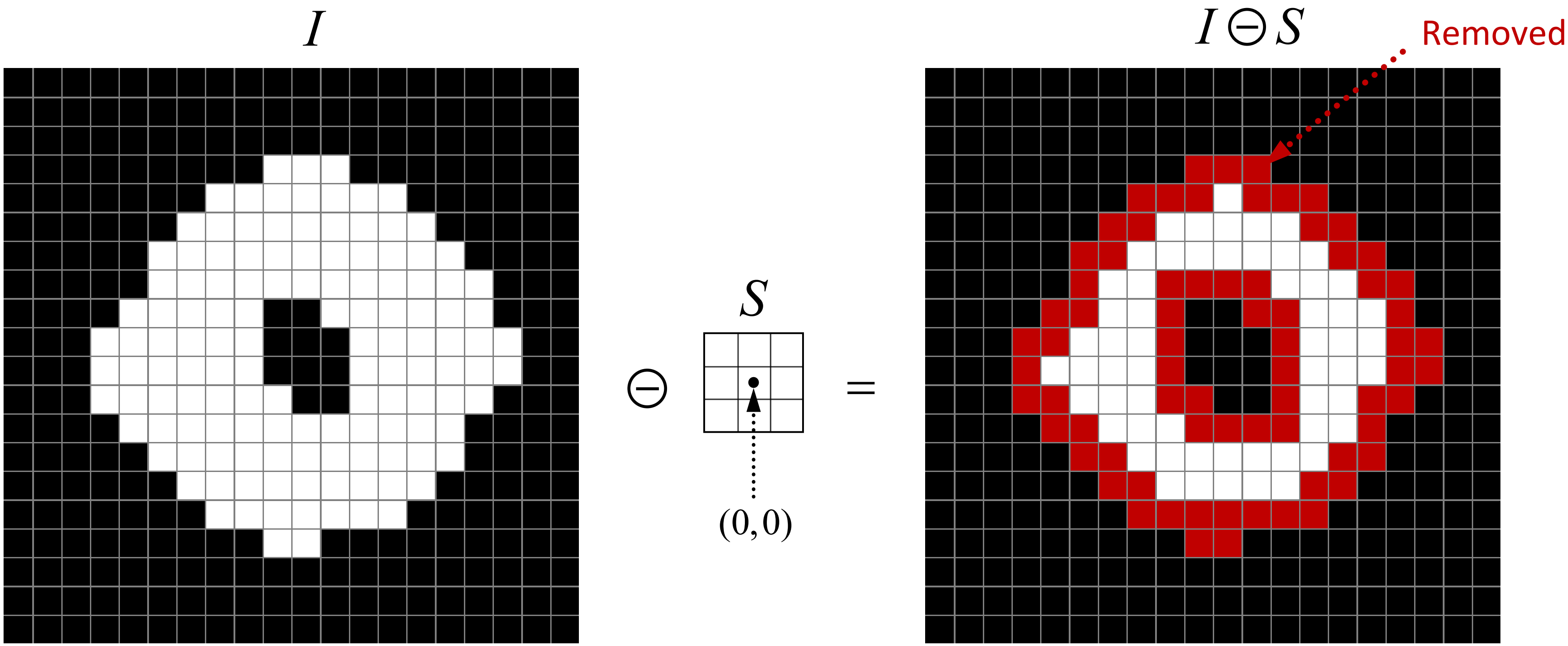
That is, the set of points $\mathbf{x} \in \mathbf{Z}^n$ for which the intersection of the image I with the reflected version of a *structuring element* S translated to $\mathbf{x}$, is not empty



Erosion of binary images

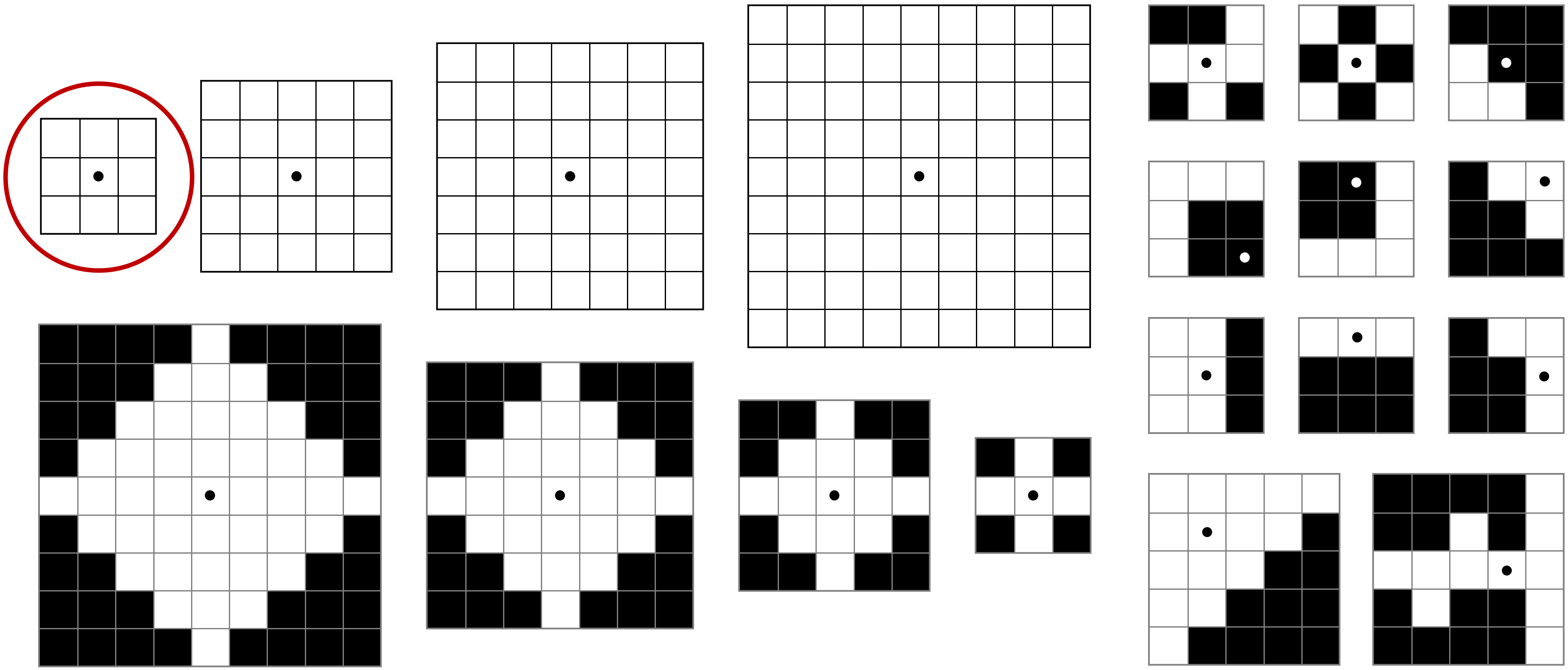
Definition of binary erosion: $I \ominus S = \{\mathbf{x} \mid S_{\mathbf{x}} \subseteq I\}$

That is, the set of points $\mathbf{x} \in \mathbf{Z}^n$ for which the *structuring element* S translated over $\mathbf{x}$ is completely contained in the image I



Structuring elements

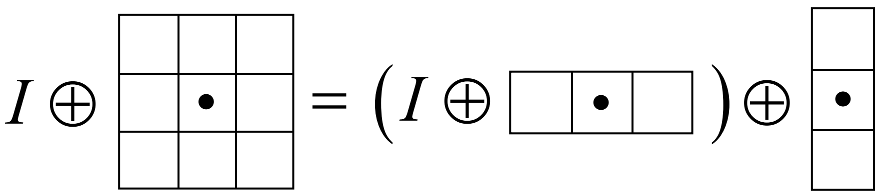
In principle a structuring element can have any shape ...



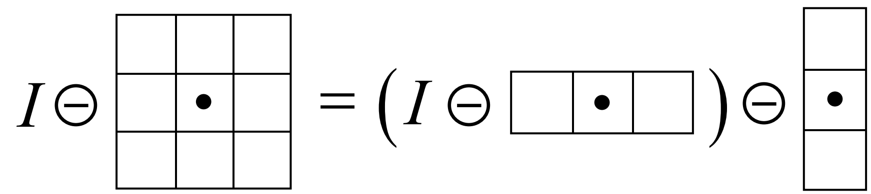
... but the symmetric 3 x 3 structuring element is used most often

Dimensional decomposition

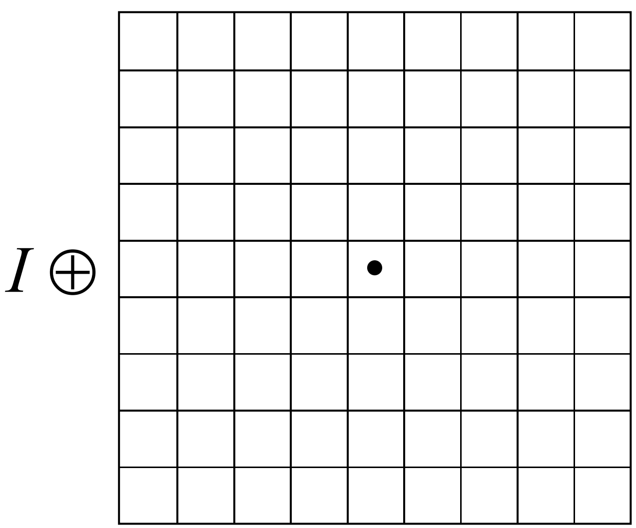
Decomposition of the basic structuring element



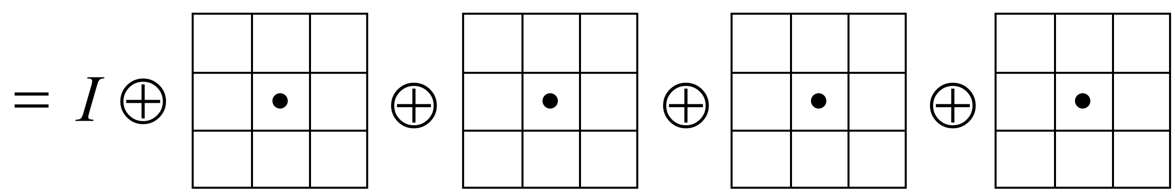
Dilation



Erosion



Iterative decomposition



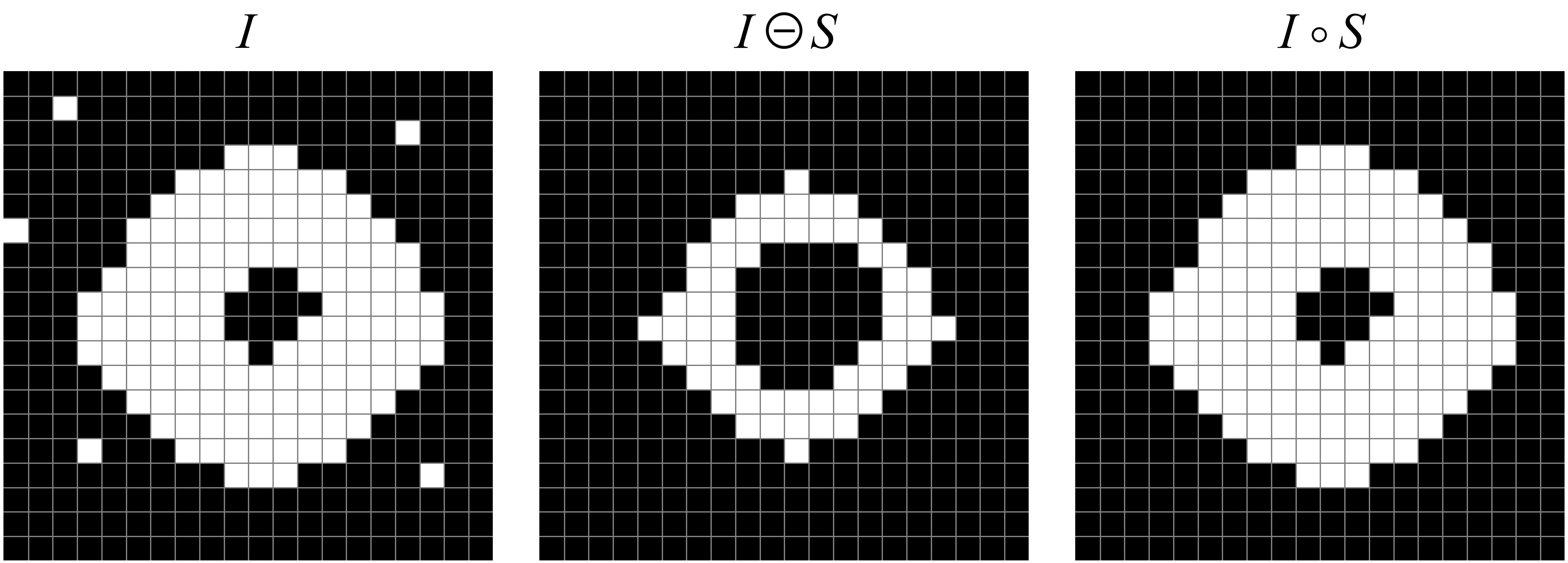
Applies to dilation and erosion

Opening of binary images

Definition of binary opening: $I \circ S = \left(I \ominus S\right) \oplus S$

That is, an erosion followed by a dilation using the same structuring element

Example using the basic 3 x 3 structuring element:



Eliminates details smaller than the structuring element *outside* the main object

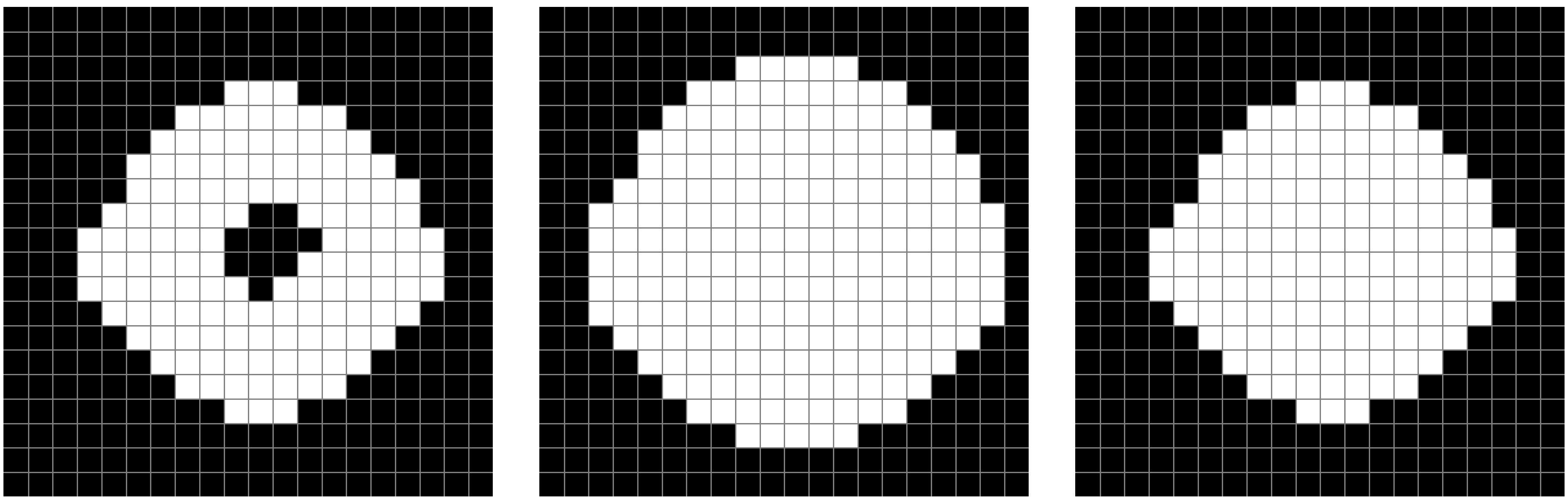
Closing of binary images

Definition of binary closing: $I \bullet S = \left(I \oplus S \right) \ominus S$

That is, a dilation followed by an erosion using the same structuring element

Example using the basic 3 x 3 structuring element:

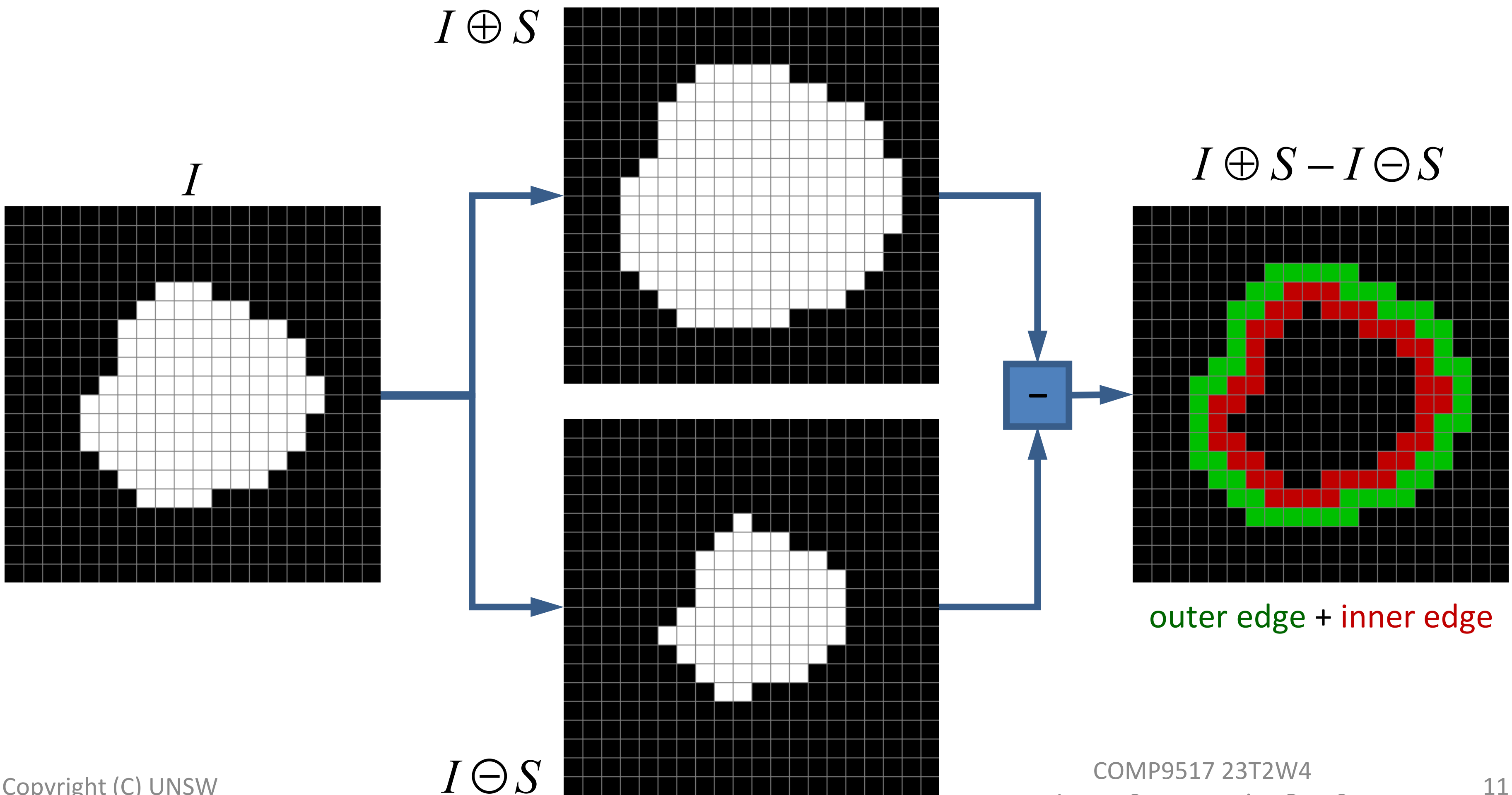
I $\qquad$ $I \oplus S$ $\qquad$ $I \bullet S$



Eliminates details smaller than the structuring element *inside* the main object

Morphological edge detection

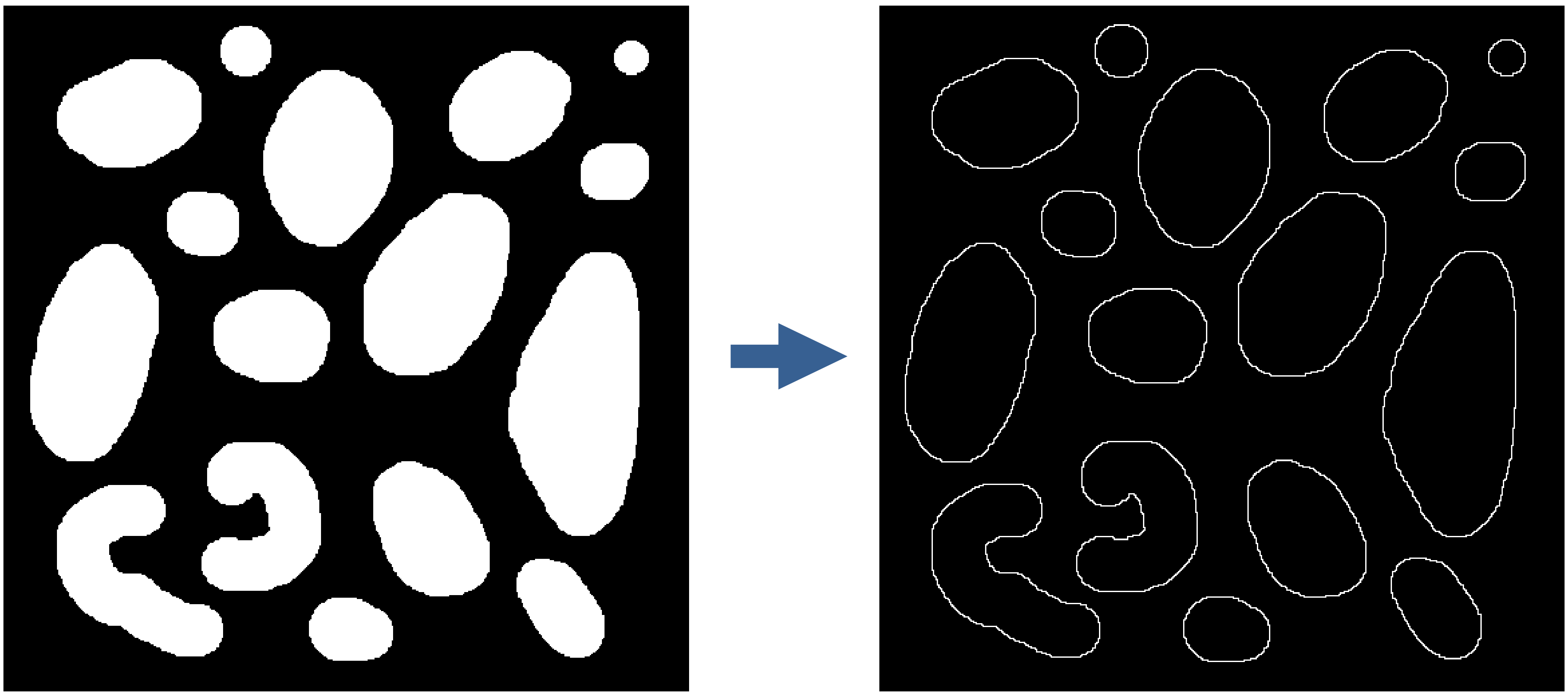
Difference between the dilated and the eroded image



Binary object outlines

How to get a one-pixel thick outline of all objects in the image?

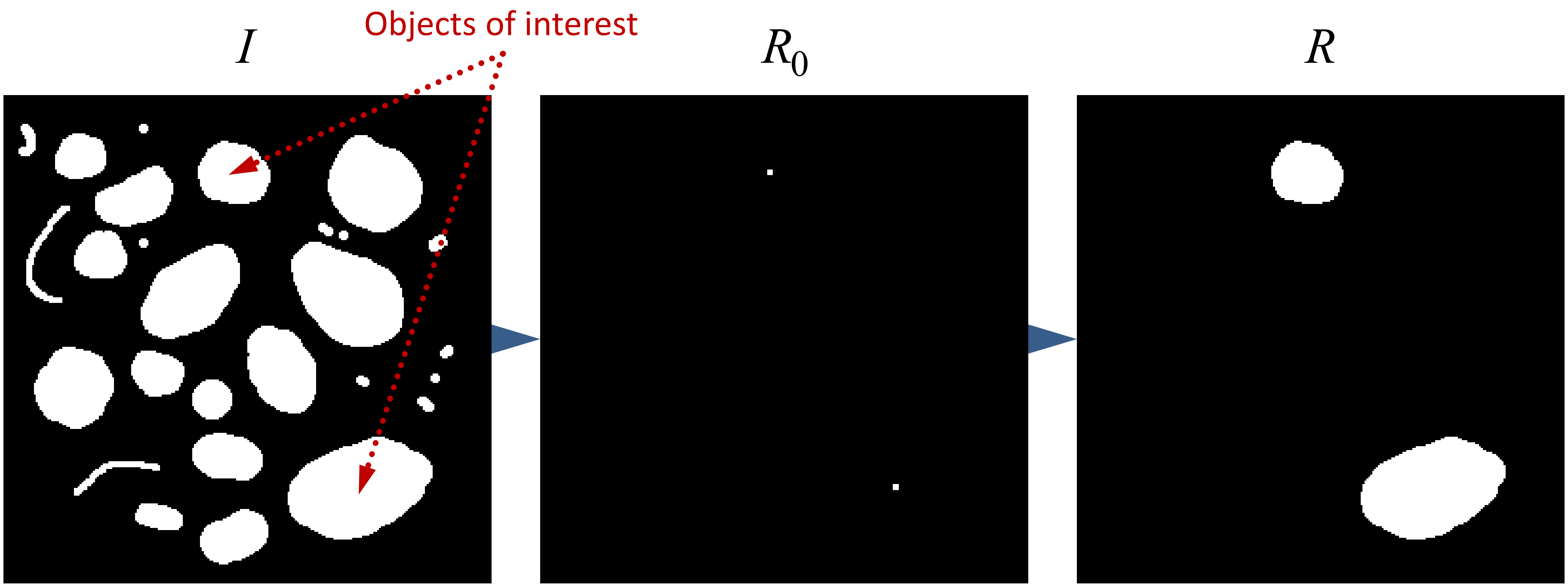
$I \qquad\qquad I \oplus S - I$



Reconstruction of binary objects

How to create an image containing selected objects only?

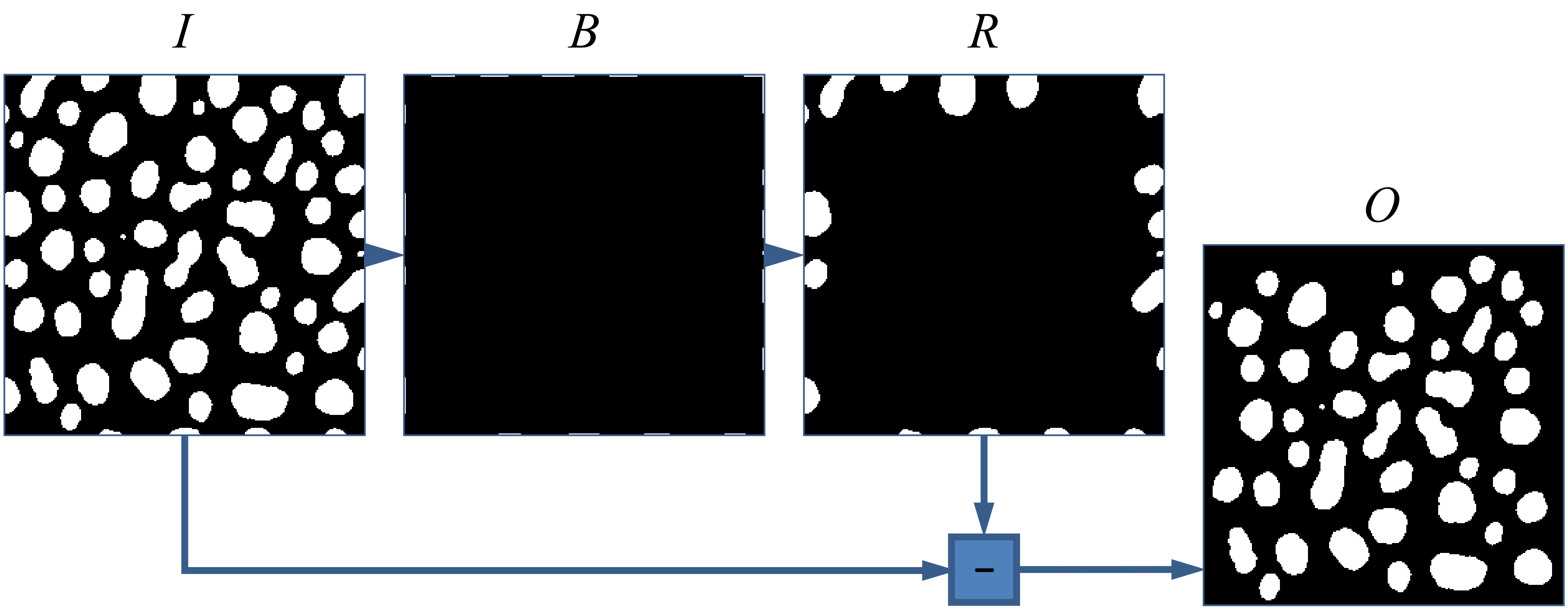
Create a marker image R_0 containing seed pixels from each selected object of image I and then iteratively compute $R_i = \left(R_{i-1} \oplus S\right) \cap I$ until $R_i = R_{i-1}$



Reconstruction of binary objects

How to remove objects that are only partly in the image?

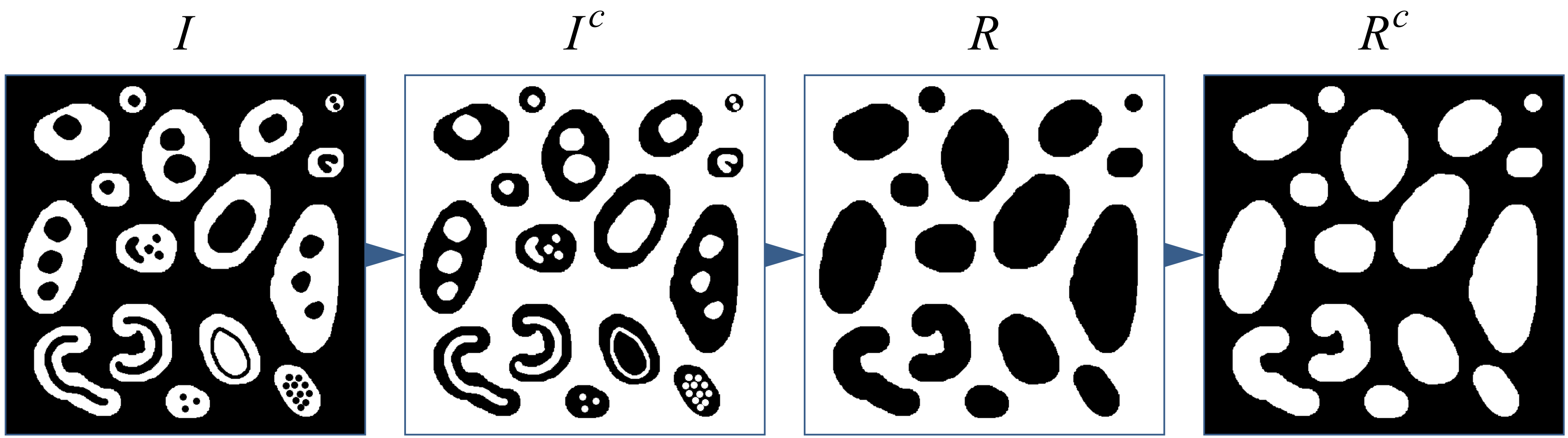
Take the boundary pixels B of input image I as the seeds, compute the reconstruction R from those seeds, and subtract the result from the input



Reconstruction of binary objects

How to fill all holes in all of the objects in the image?

Take the complement I^c of image I, take the boundary pixels of I^c as seeds, compute reconstruction R of I^c from those seeds, take the complement R^c

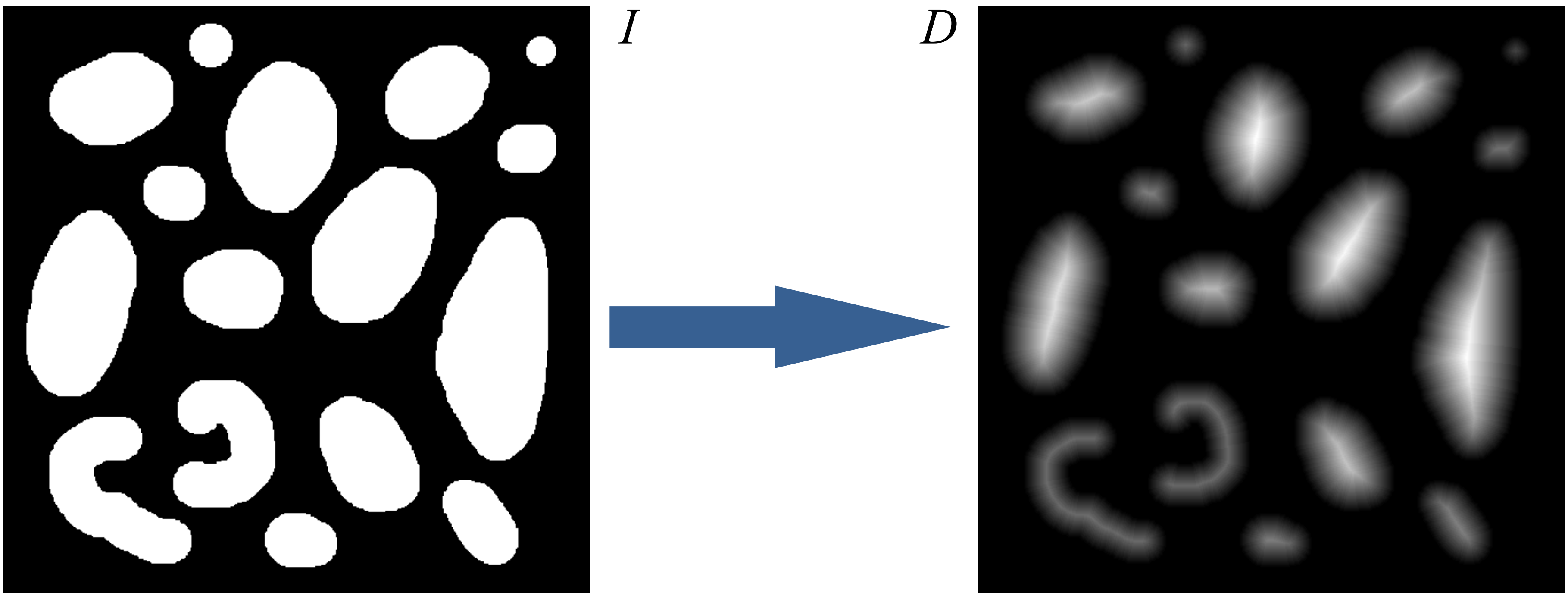


Distance transform of binary images

How to compute the distance of object pixels to the background?

Denote input image I as I_0 and iteratively compute $I_i = I_{i-1} \ominus S$ for $i = 1, 2, \ldots$

while setting all pixels eroded in iteration i to value i in the output image D

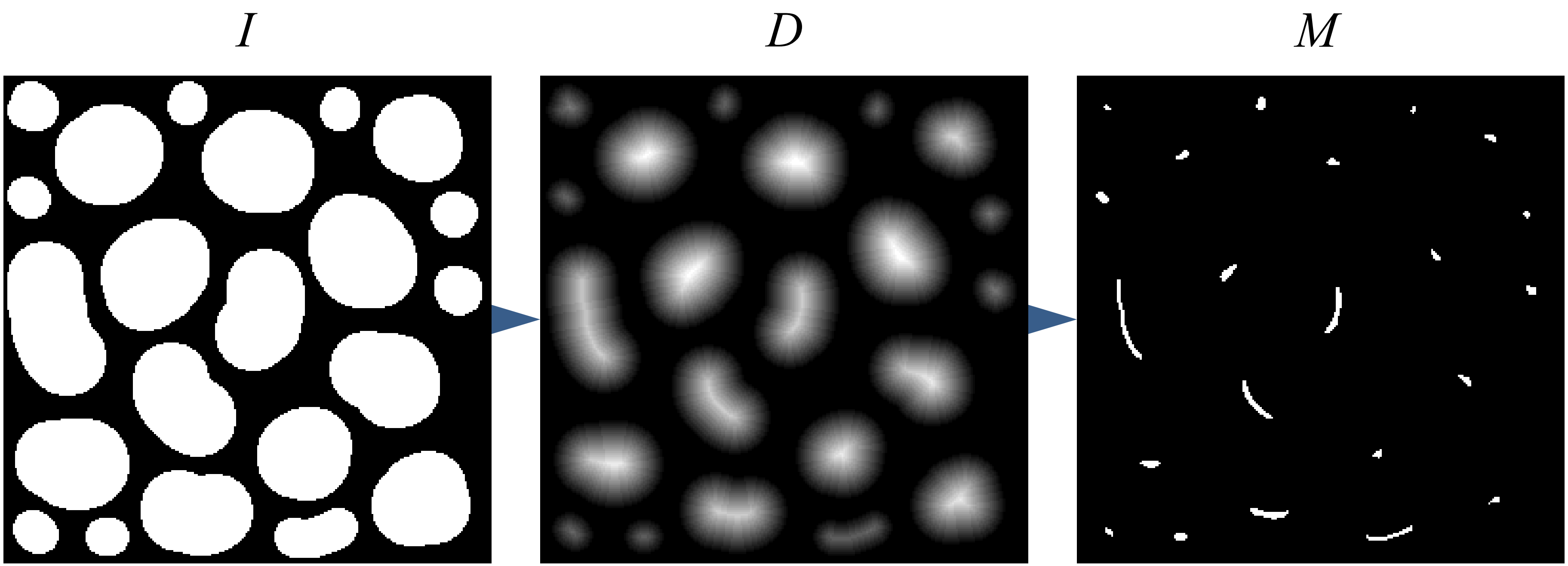


Ultimate erosion of binary images

How to find representative center points for all the objects?

Compute the distance transform of the image and find all the local maxima

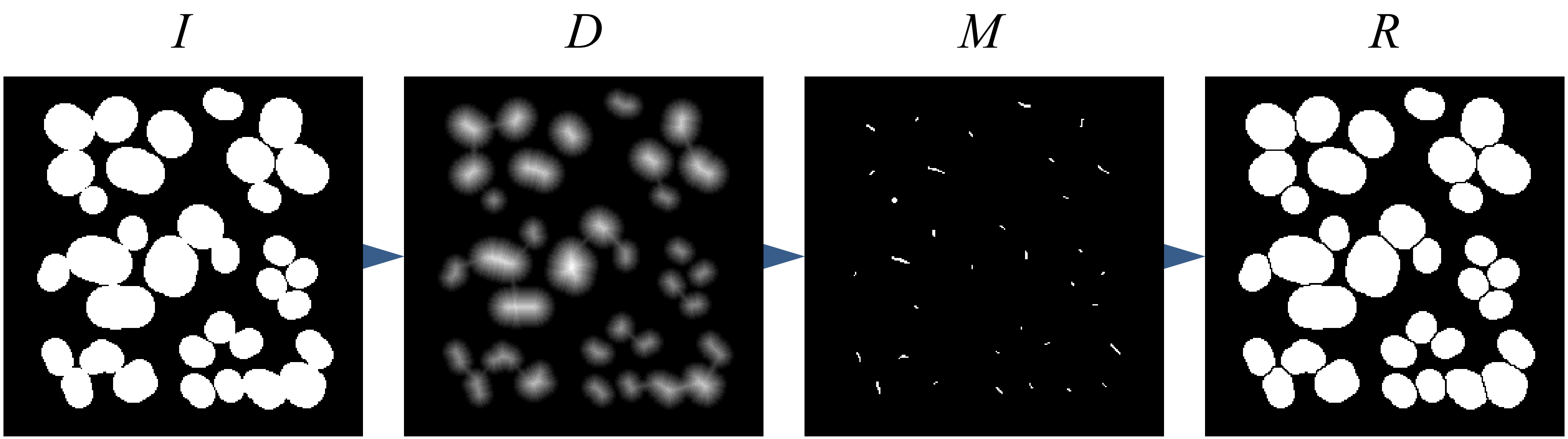
This is the same as keeping only the last object pixels before final erosion



Ultimate erosion and reconstruction

How to separate objects that are touching each other?

Perform ultimate erosion and then perform a reconstruction of the result with the additional constraint that objects may not merge



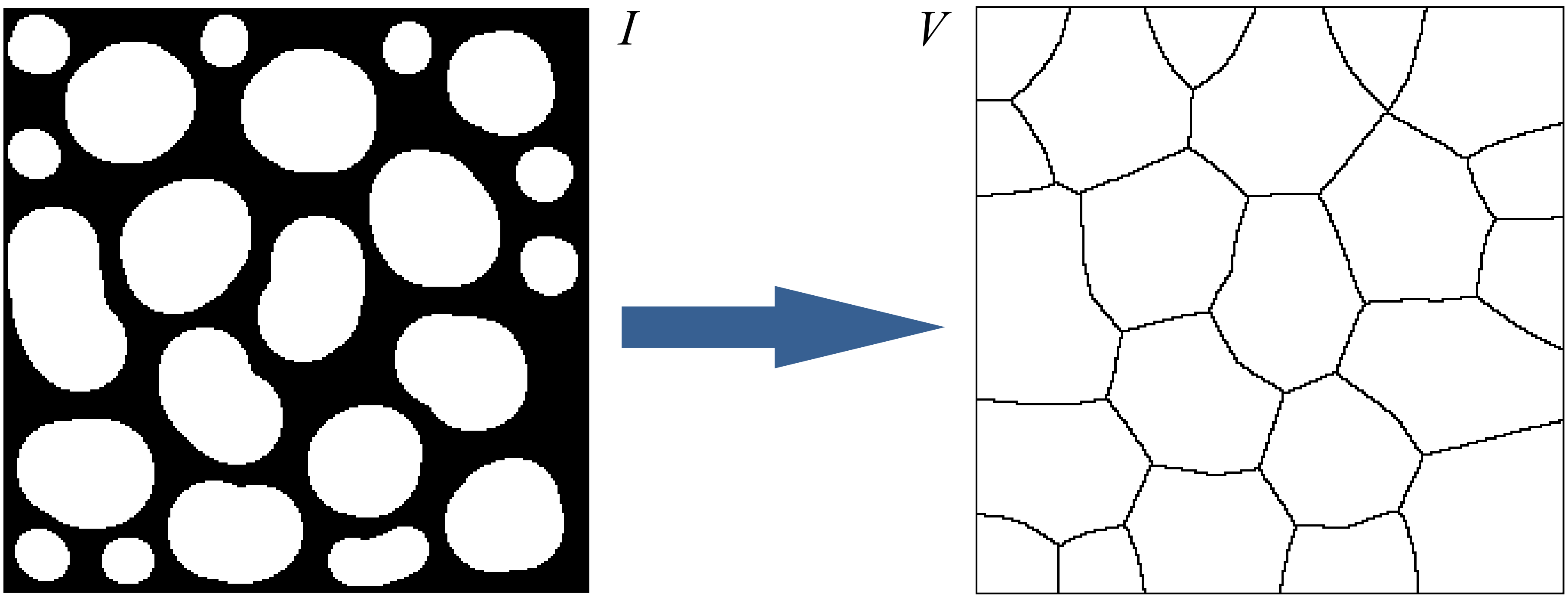
Since elongated objects may result in multiple local maxima this approach works best for objects that are more or less circular

Ultimate dilation of binary images

How to find the background points equidistant to the objects?

Iteratively dilate the image while imposing the non-merging constraint

The result is called the Voronoi (or Dirichlet) tessellation of the objects

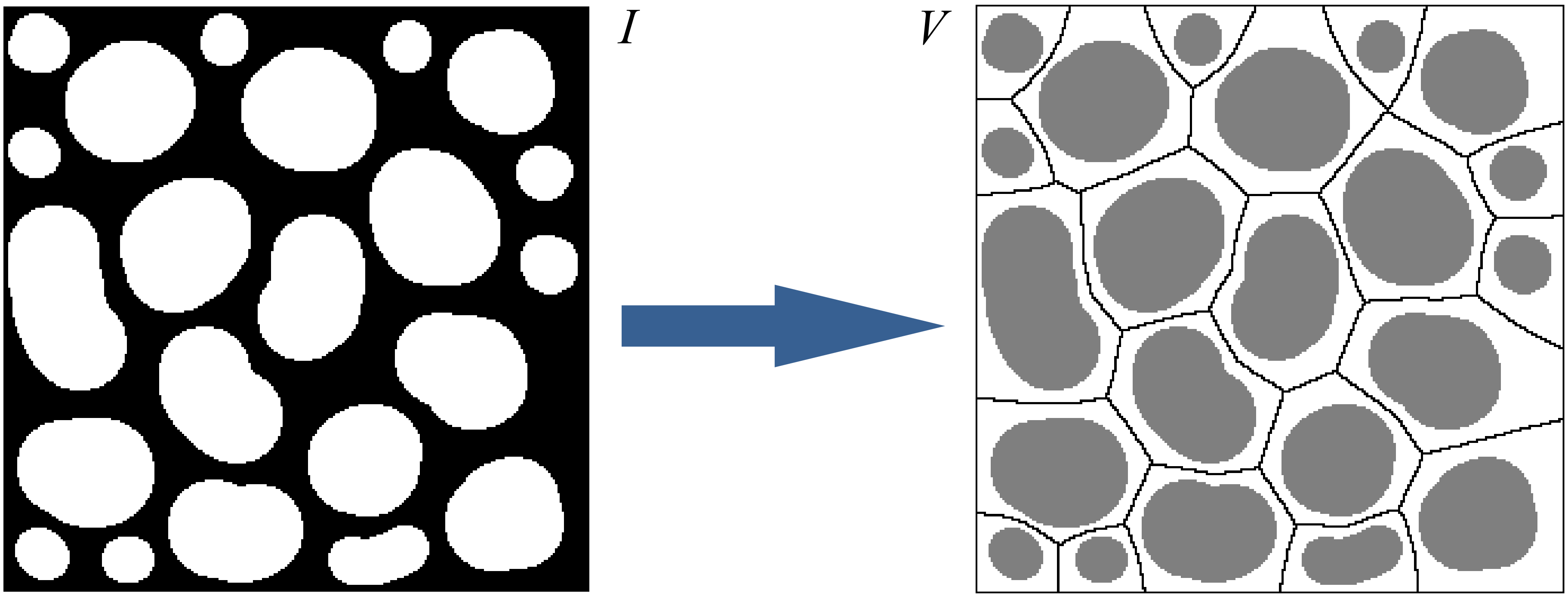


Ultimate dilation of binary images

How to find the background points equidistant to the objects?

Iteratively dilate the image while imposing the non-merging constraint

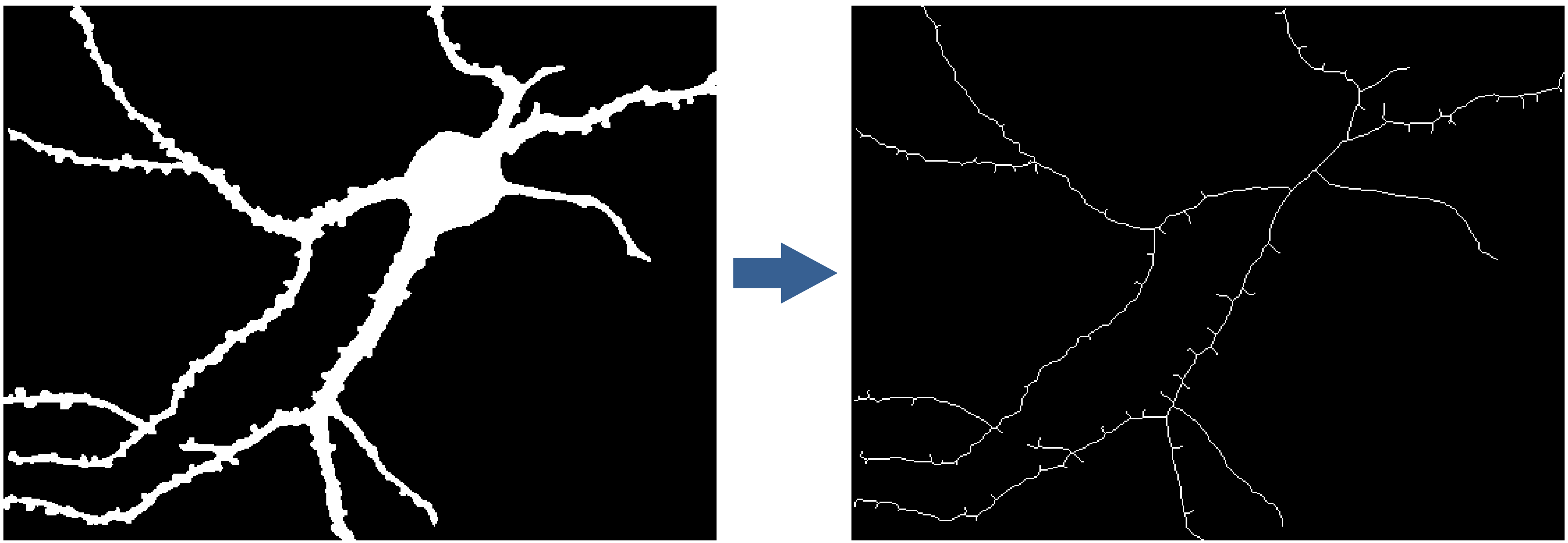
The result is called the Voronoi (or Dirichlet) tessellation of the objects



Skeletonization of binary images

How to find a representative centerline of the objects?

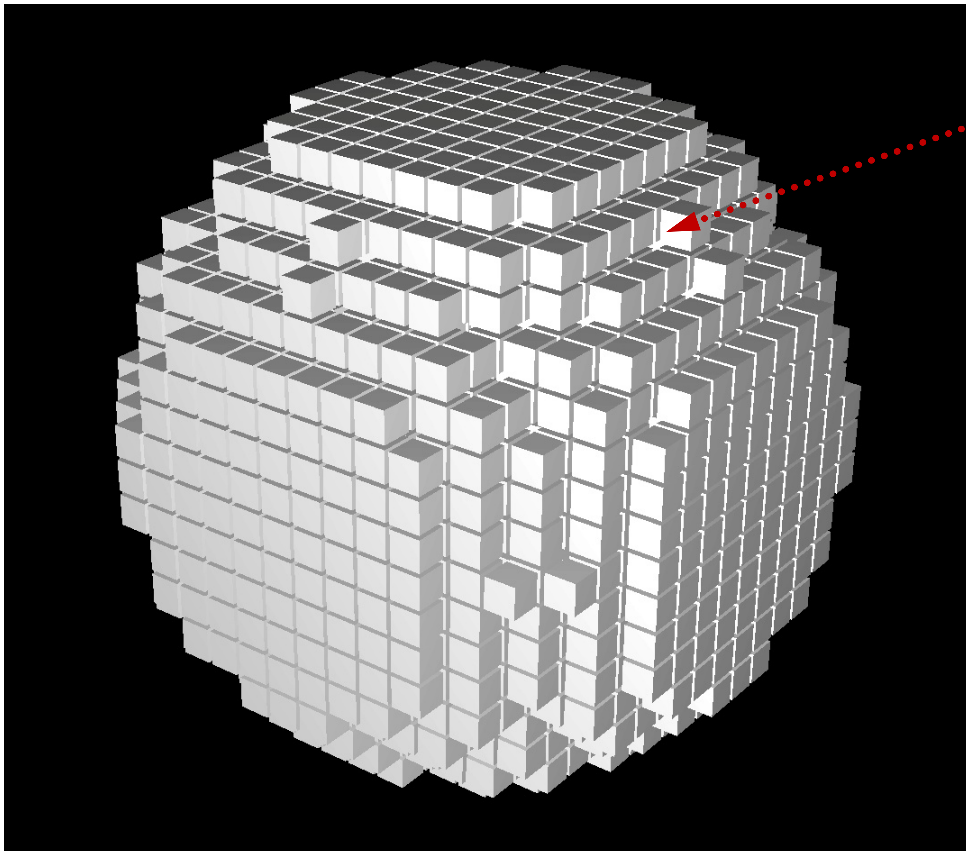
Iteratively apply conditional erosion (thinning) that does not break the connectivity of the result and does not remove single pixels or end-pixels



The resulting one-pixel thick structure is called the *skeleton* of the object

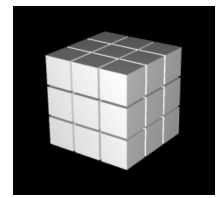
Binary morphology of *n*D images

The presented concepts extend to any dimensionality



Example of a 3D binary image

Volumetric pixels ("voxels")

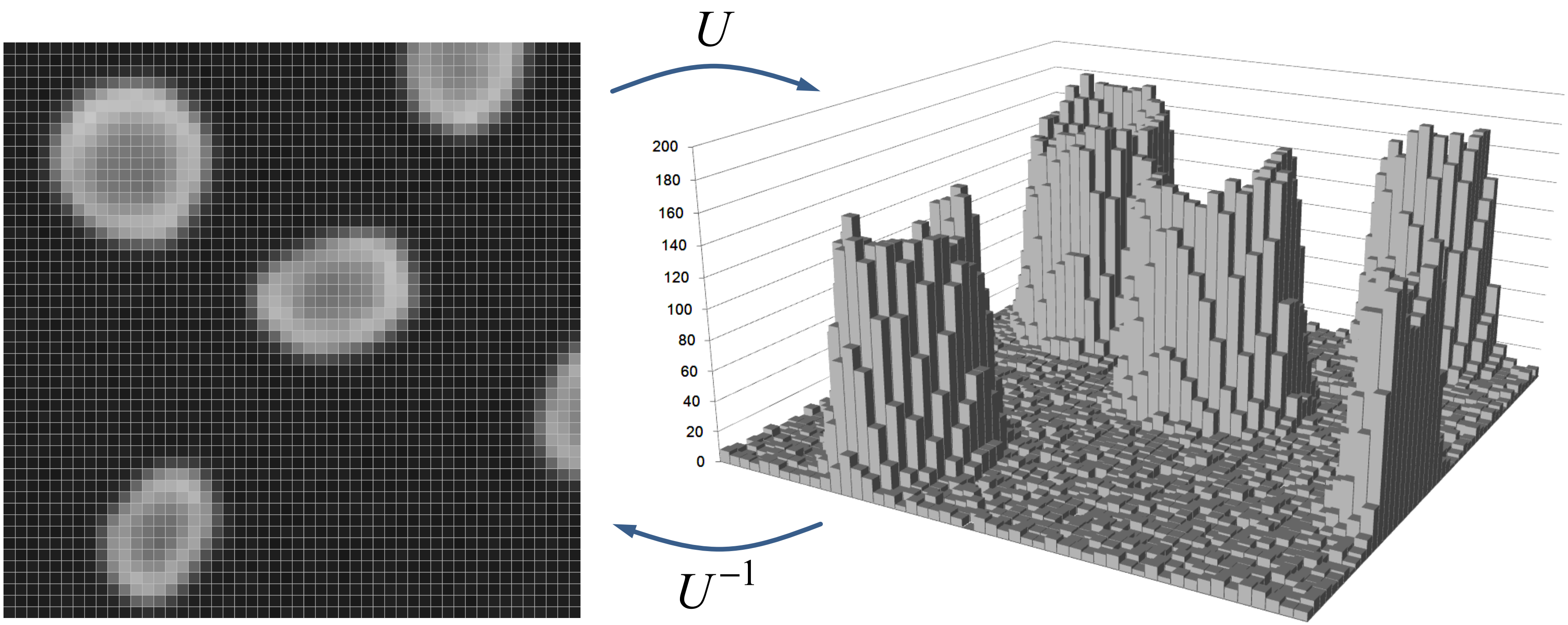


3 x 3 x 3 voxel
structuring
element

- 3D dilation
- 3D erosion
- 3D opening
- 3D closing
- And all algorithms based on it

Gray-scale mathematical morphology

Consider *n*D gray-scale images as (*n*+1)D binary images…



2D gray-scale image I 3D binary image $U(I)$

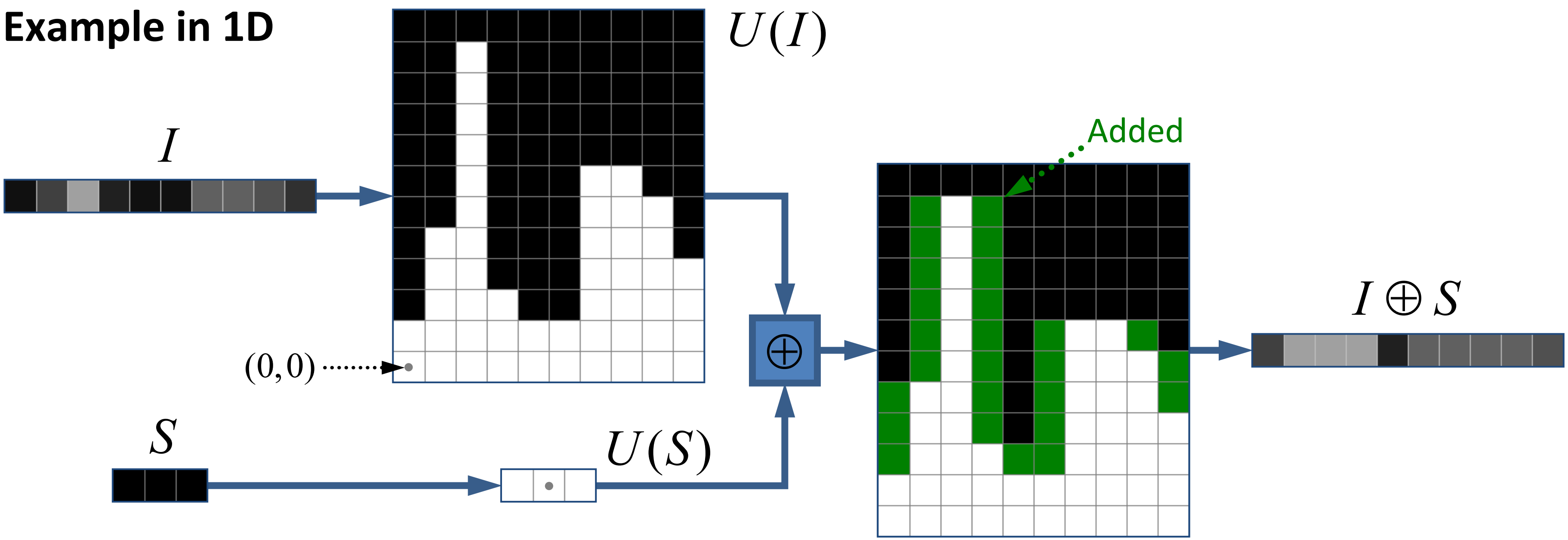
The landscape surface with the volume below is called the *umbra* of the image

Dilation of gray-scale images

Definition of gray-scale dilation: $I \oplus S = U^{-1}\left[U(I) \oplus U(S)\right]$

That is, the *binary dilation* of the umbra $U(I)$ of *gray-scale image* I with the umbra $U(S)$ of *gray-scale structuring element* S, turned back into gray-scale

Example in 1D

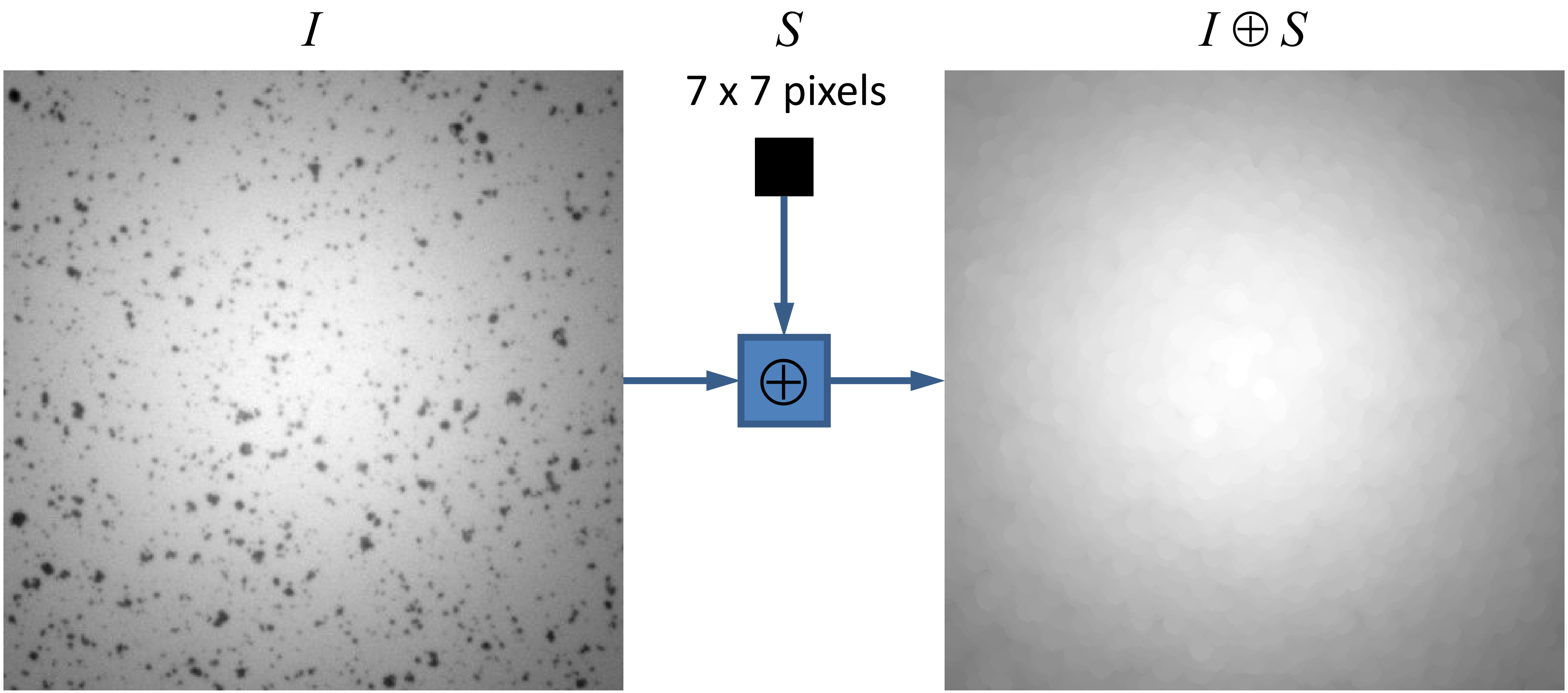


Any gray-scale S is possible but the flat one (as shown here) is used most often

Dilation of gray-scale images

Equivalent definition: $(I \oplus S)(\mathbf{x}) = \max_{\mathbf{p}} \{I(\mathbf{x}-\mathbf{p}) + S(\mathbf{p})\}$

For a flat and symmetrical structuring element this is simply local max-filtering

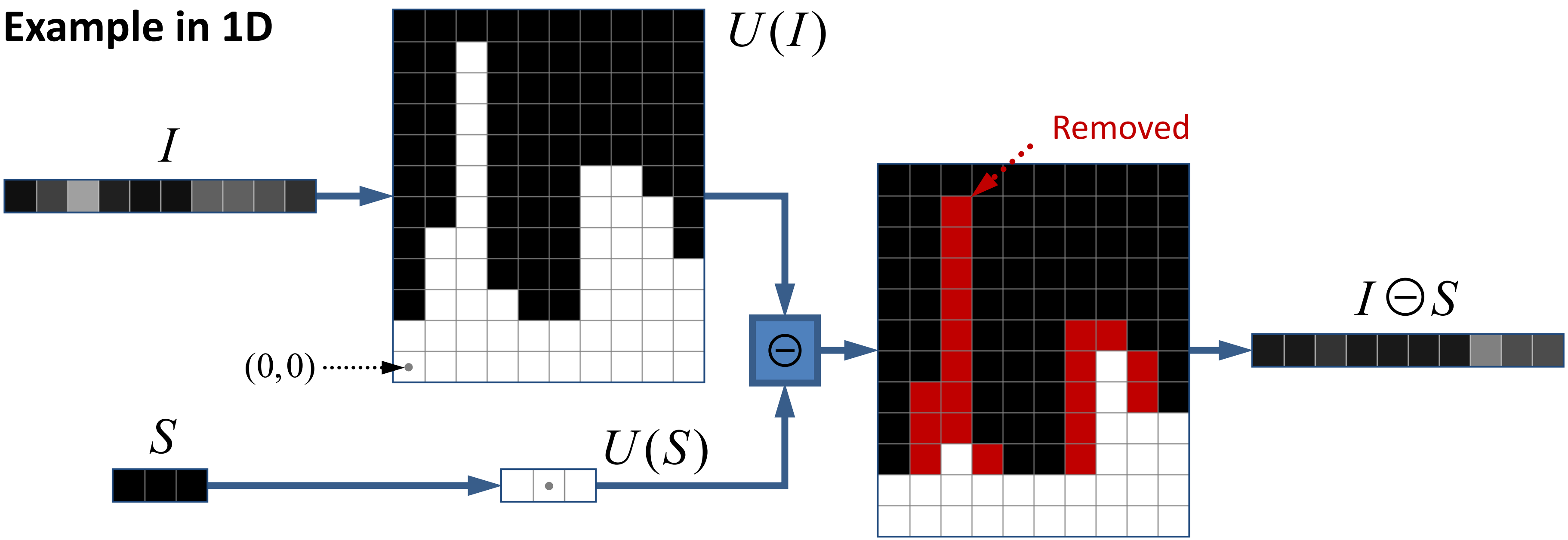


Erosion of gray-scale images

Definition of gray-scale erosion: $I \ominus S = U^{-1}\left[U(I) \ominus U(S)\right]$

That is, the *binary erosion* of the umbra $U(I)$ of *gray-scale image* I with the umbra $U(S)$ of *gray-scale structuring element* S, turned back into gray-scale

Example in 1D

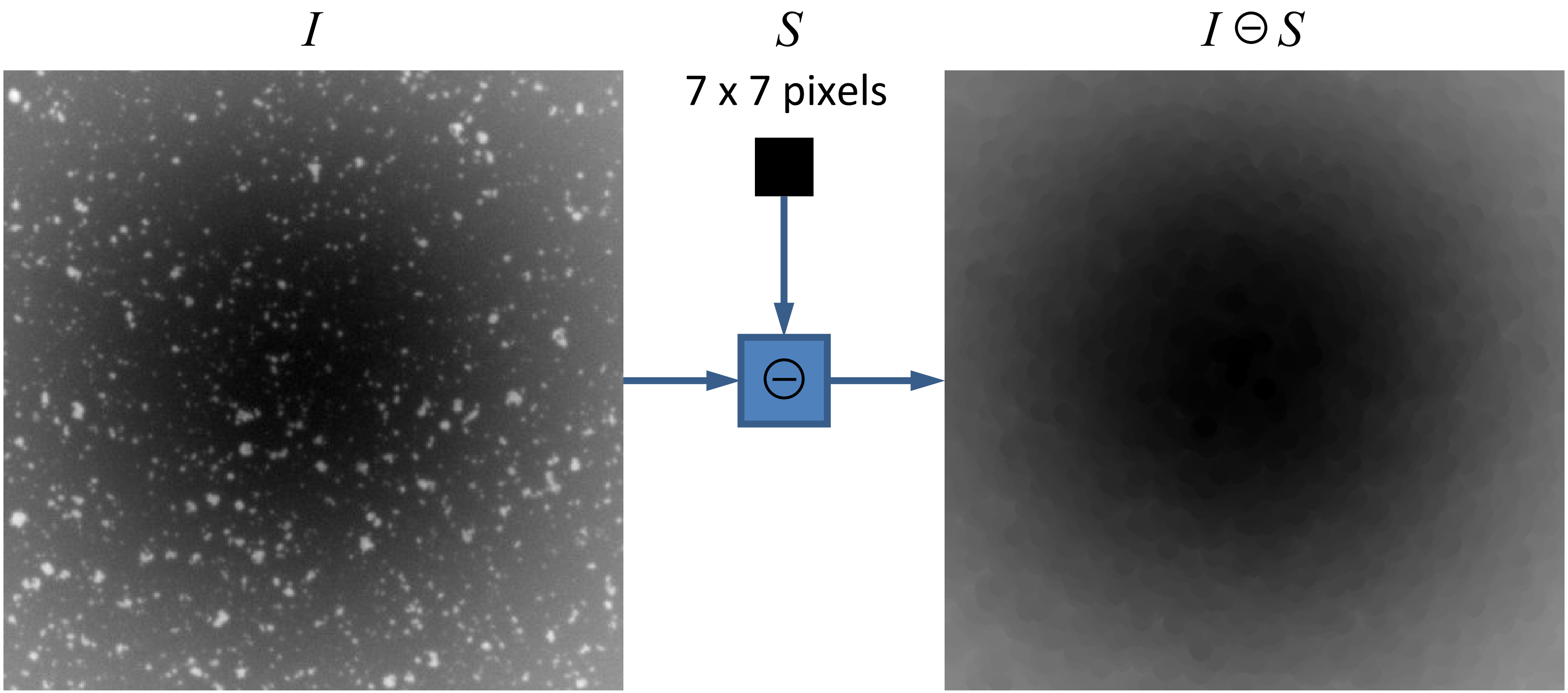


Any gray-scale S is possible but the flat one (as shown here) is used most often

Erosion of gray-scale images

Equivalent definition: $(I \ominus S)(\mathbf{x}) = \min_{\mathbf{p}} \{I(\mathbf{x}+\mathbf{p}) - S(\mathbf{p})\}$

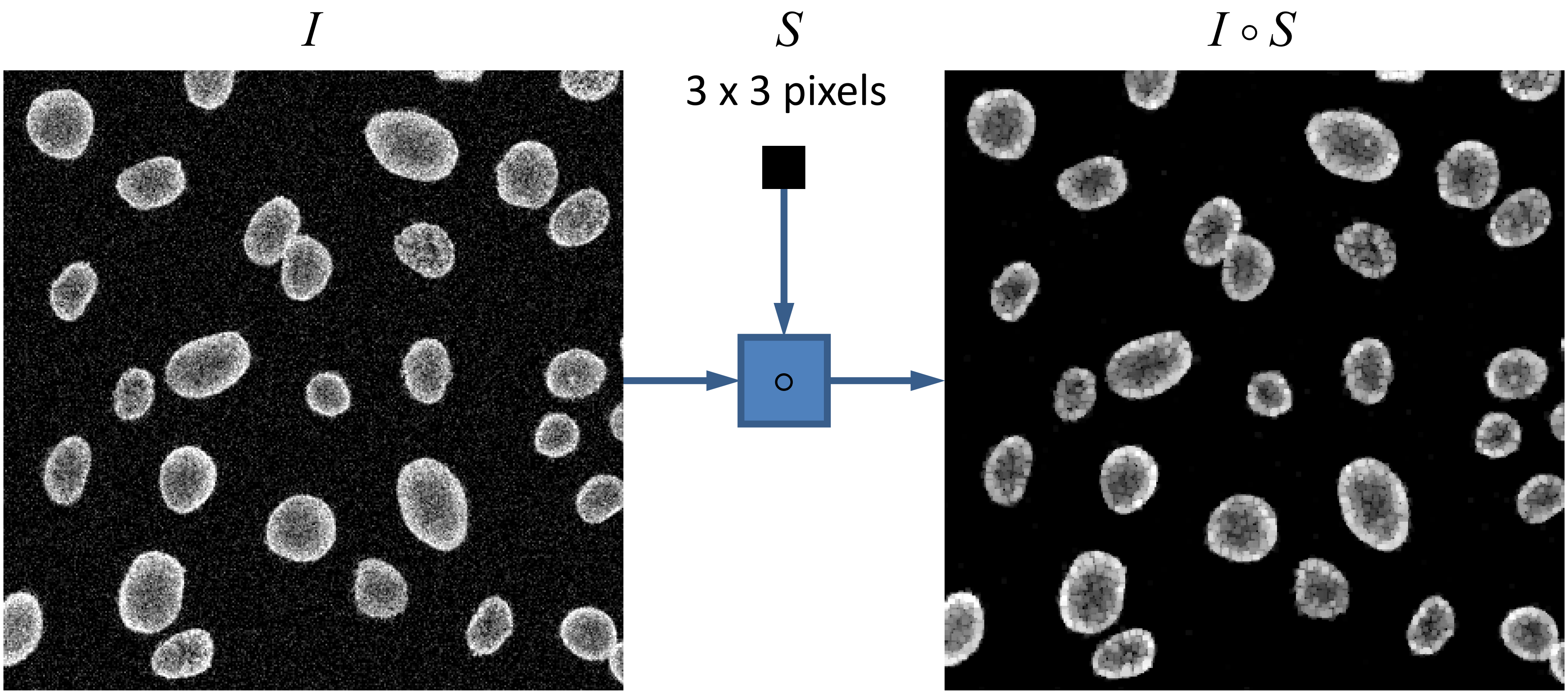
For a flat and symmetrical structuring element this is simply local min-filtering



Opening of gray-scale images

Definition of gray-scale opening: $I \circ S = (I \ominus S) \oplus S$

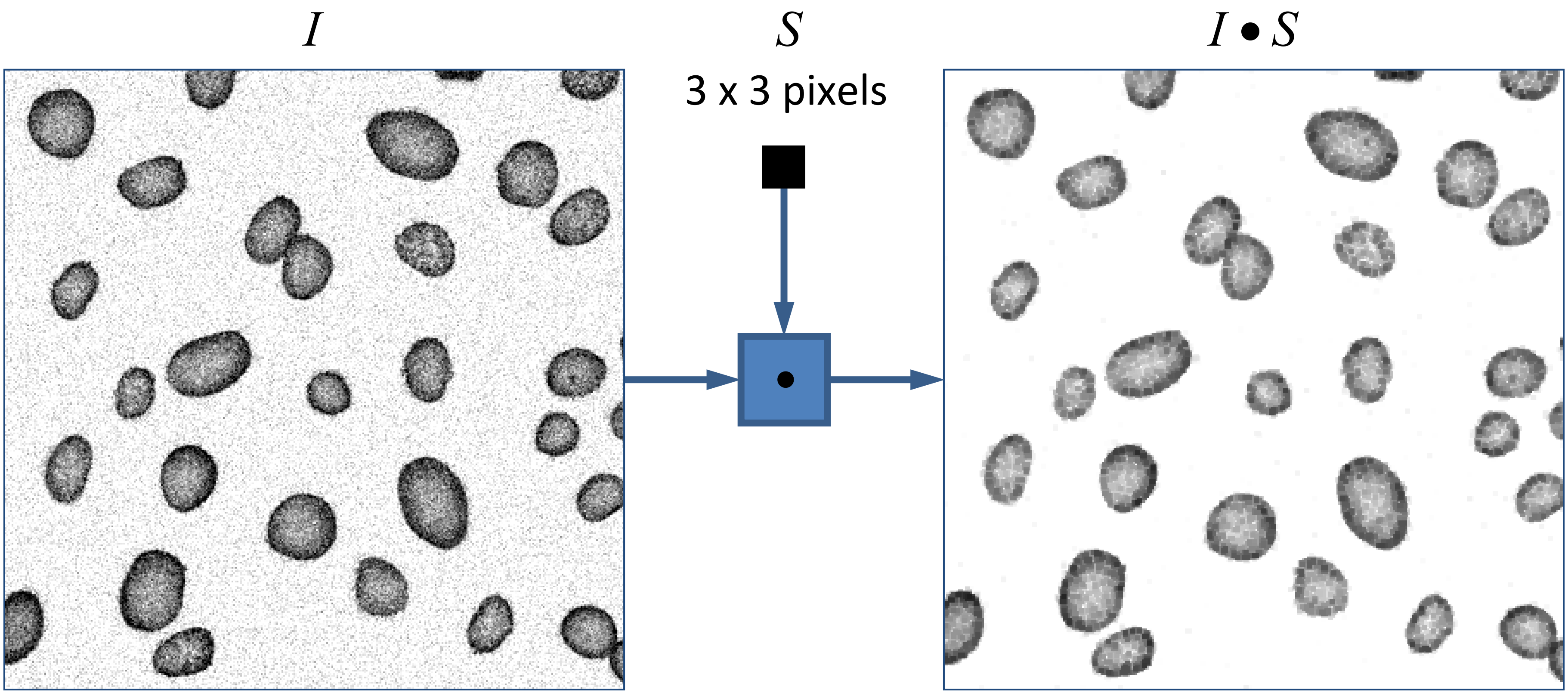
Gray-scale erosion and then gray-scale dilation with same structuring element



Closing of gray-scale images

Definition of gray-scale closing: $I \bullet S = (I \oplus S) \ominus S$

Gray-scale dilation and then gray-scale erosion with same structuring element

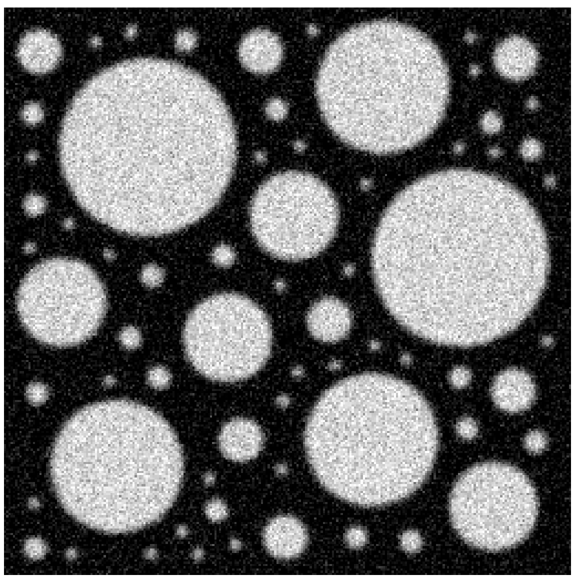


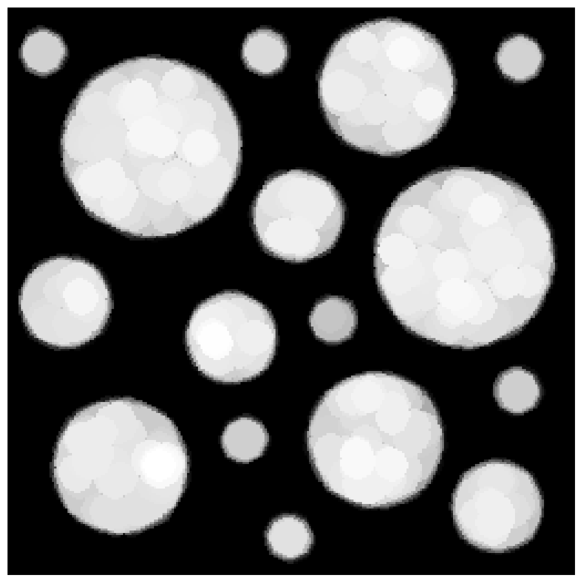
Morphological smoothing of gray-scale images

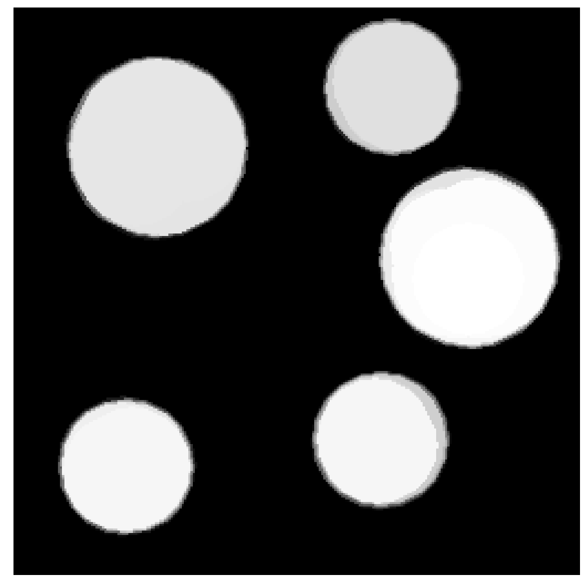
Suppressing image structures of specific size (and shape)

- High-valued (bright) image structures are removed by gray-scale opening
- Low-valued (dark) image structures are removed by gray-scale closing

I | $I \circ S$ (radius = 7 pixels) | $I \circ S$ (radius = 25 pixels)



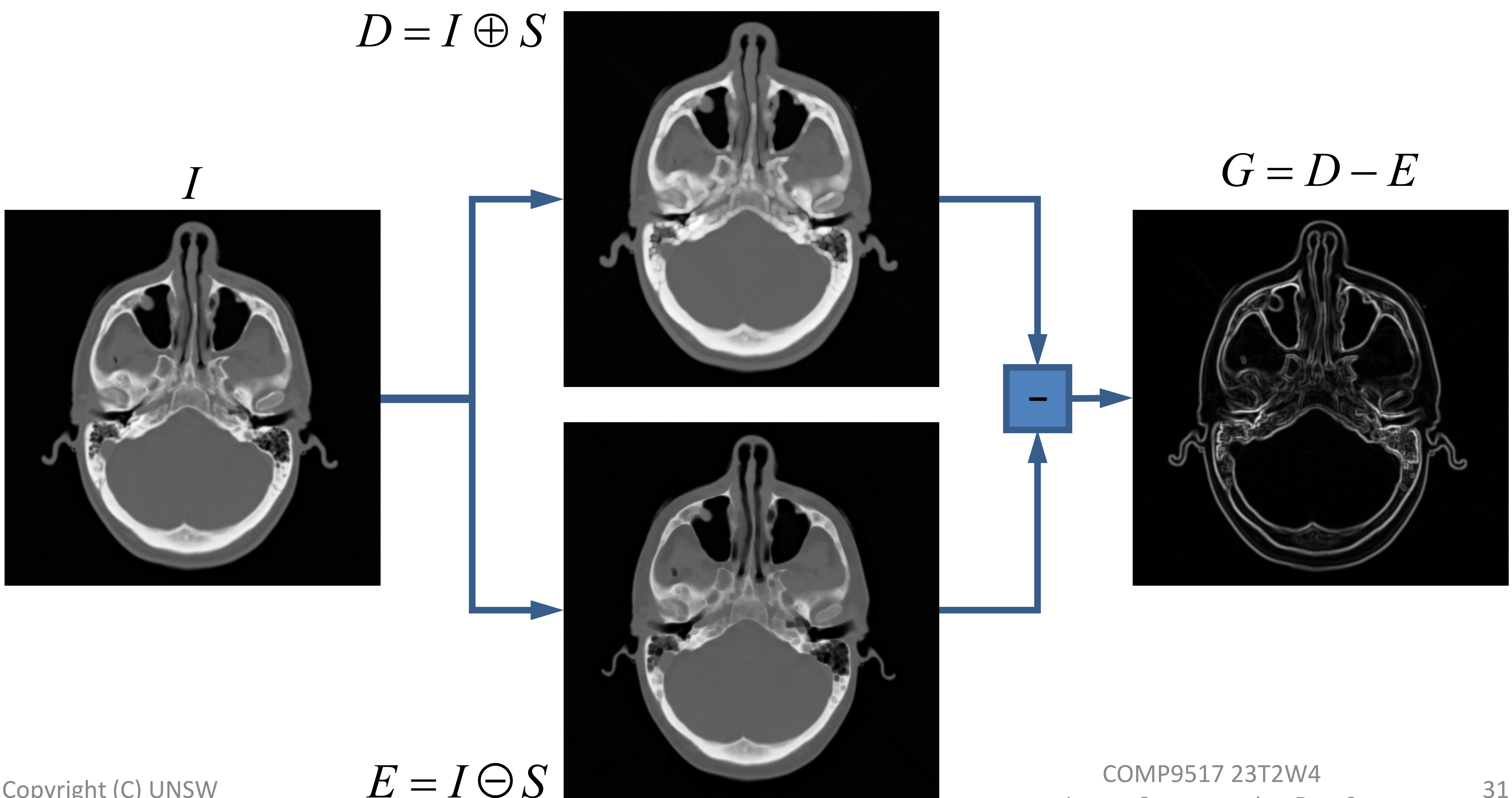




Behold the power of nonlinear filtering (not possible with linear filtering)

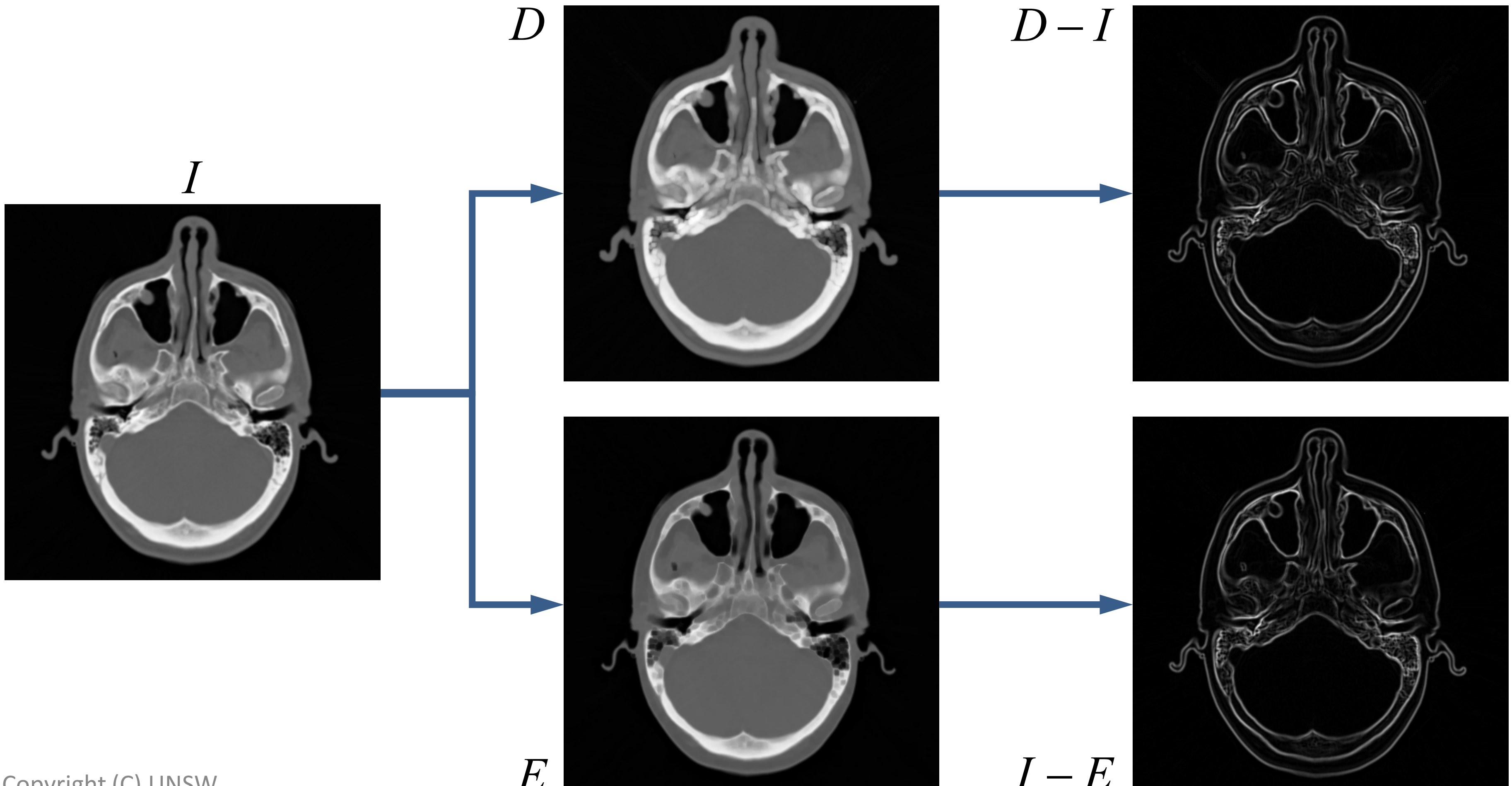
Morphological gradient of gray-scale images

Difference between the dilated and the eroded image



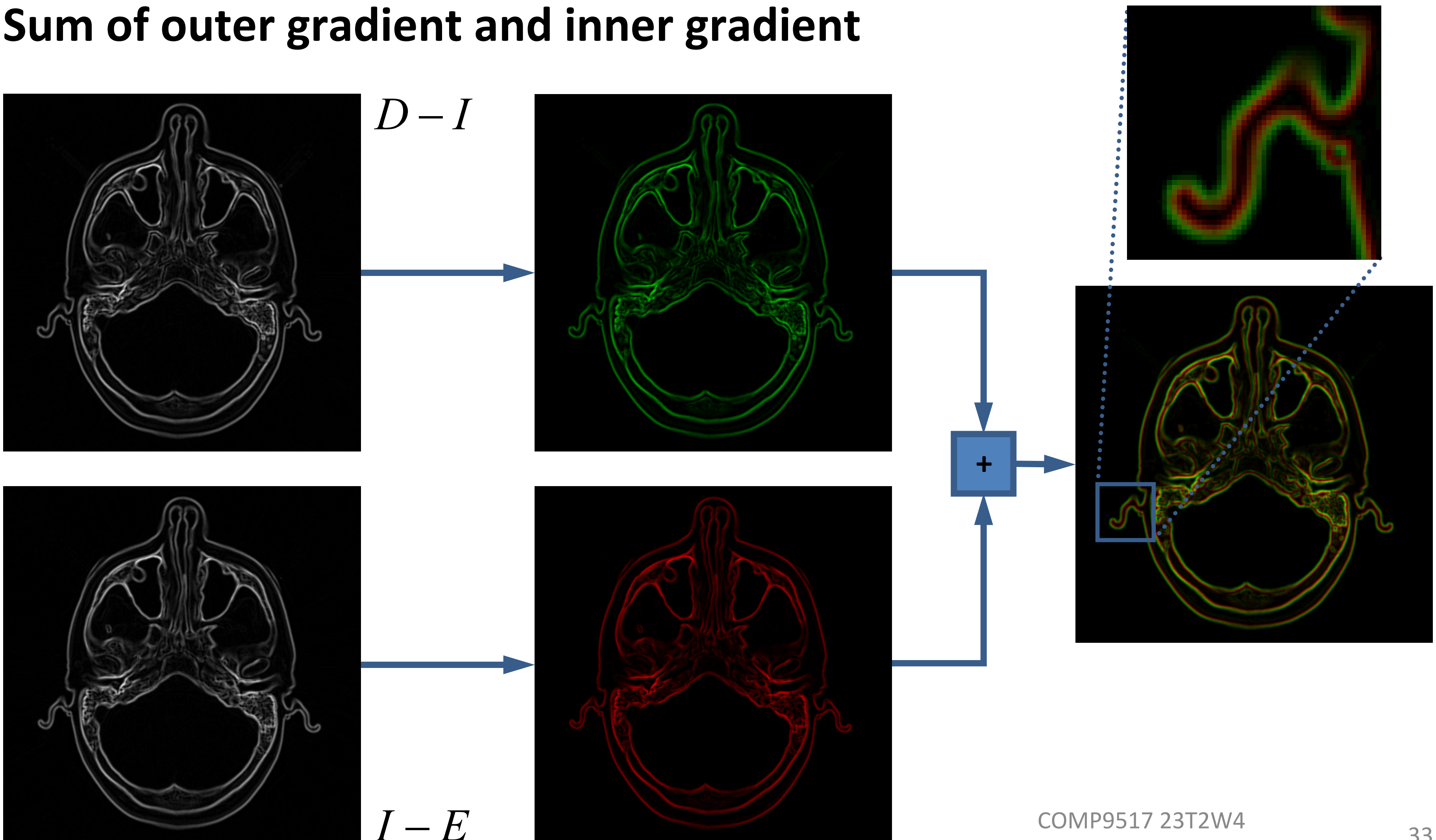
Morphological gradient of gray-scale images

Outer gradient and inner gradient



Morphological gradient of gray-scale images

Sum of outer gradient and inner gradient

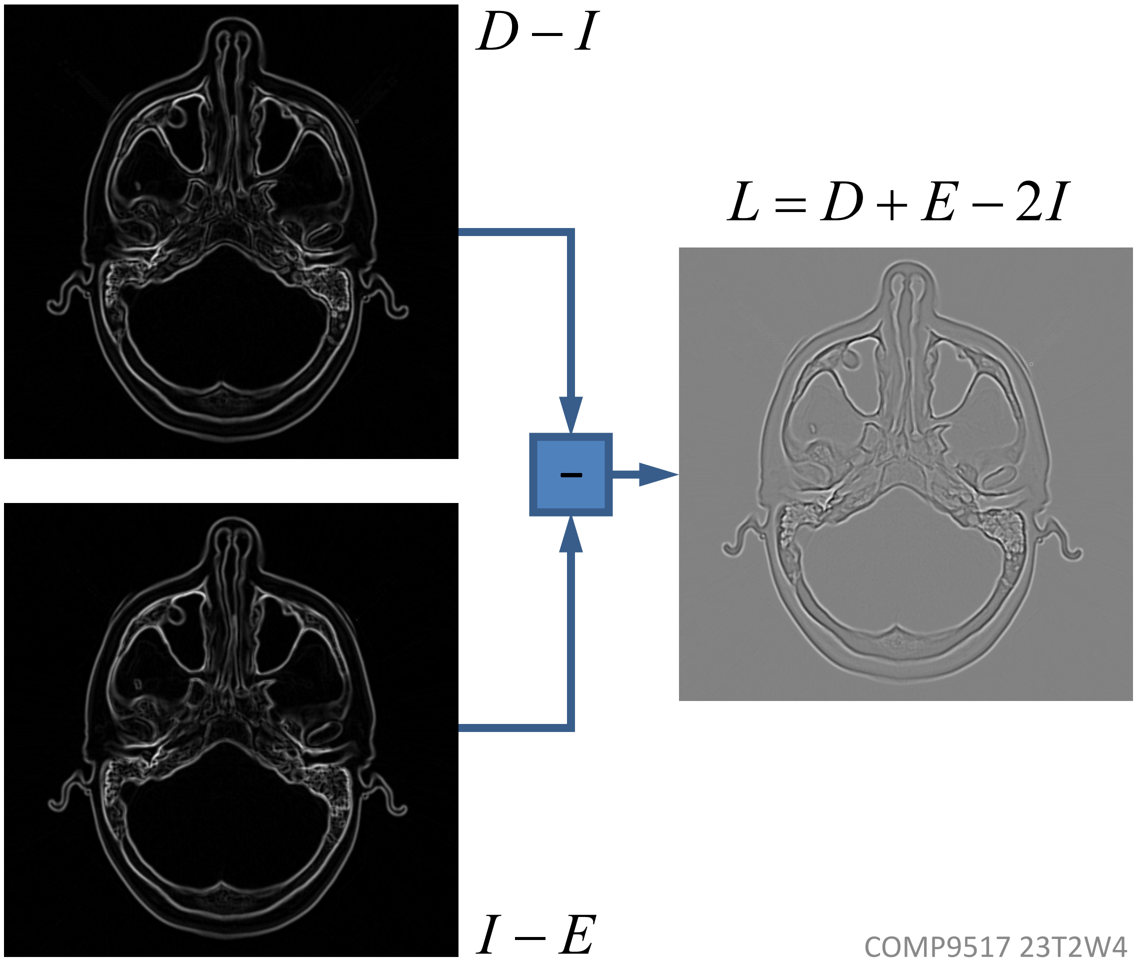


Morphological Laplacean of gray-scale images

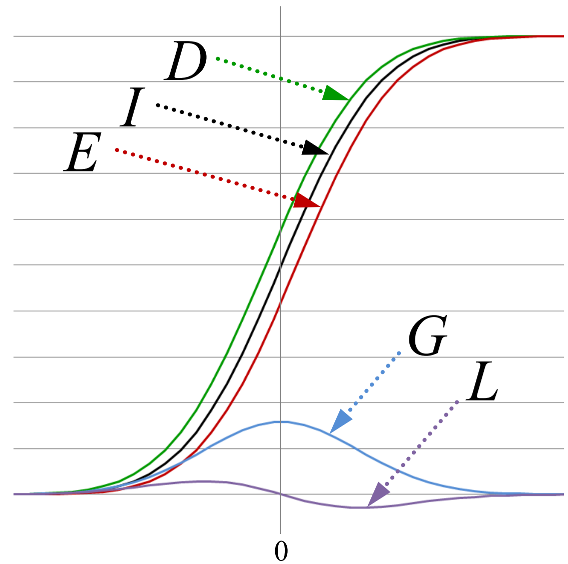
Difference between outer gradient and inner gradient

$D - I$

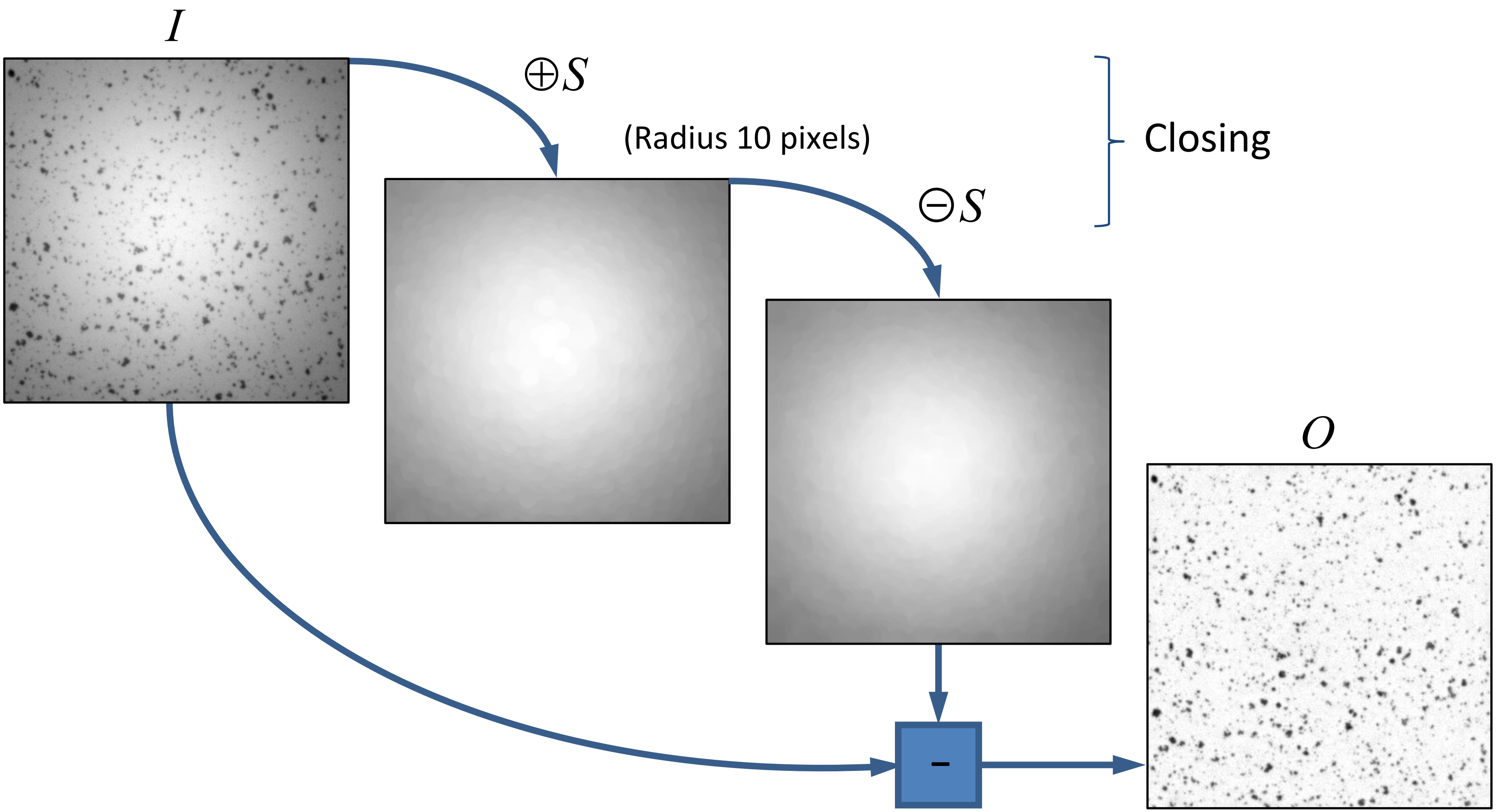
$L = D + E - 2I$



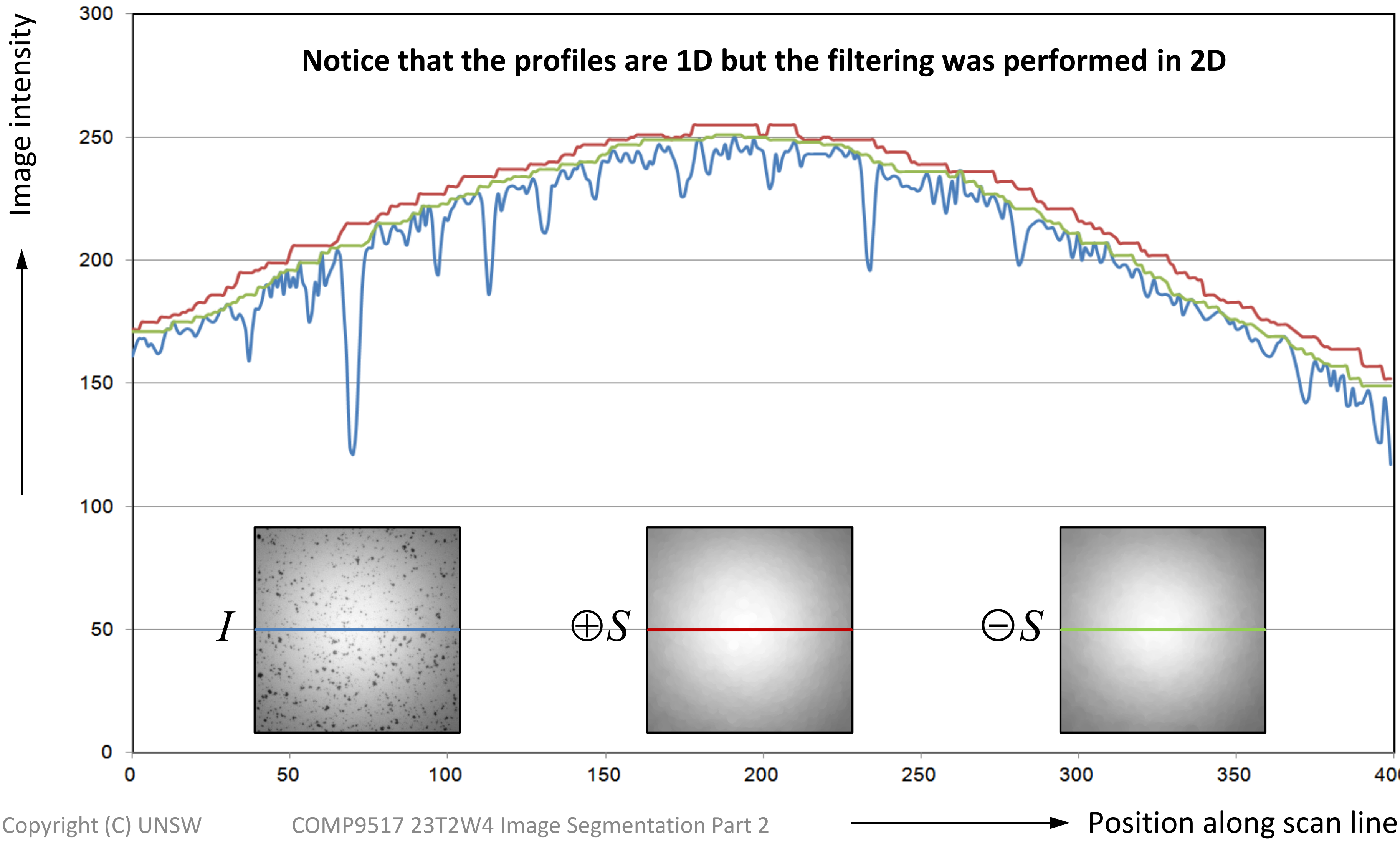
$I - E$



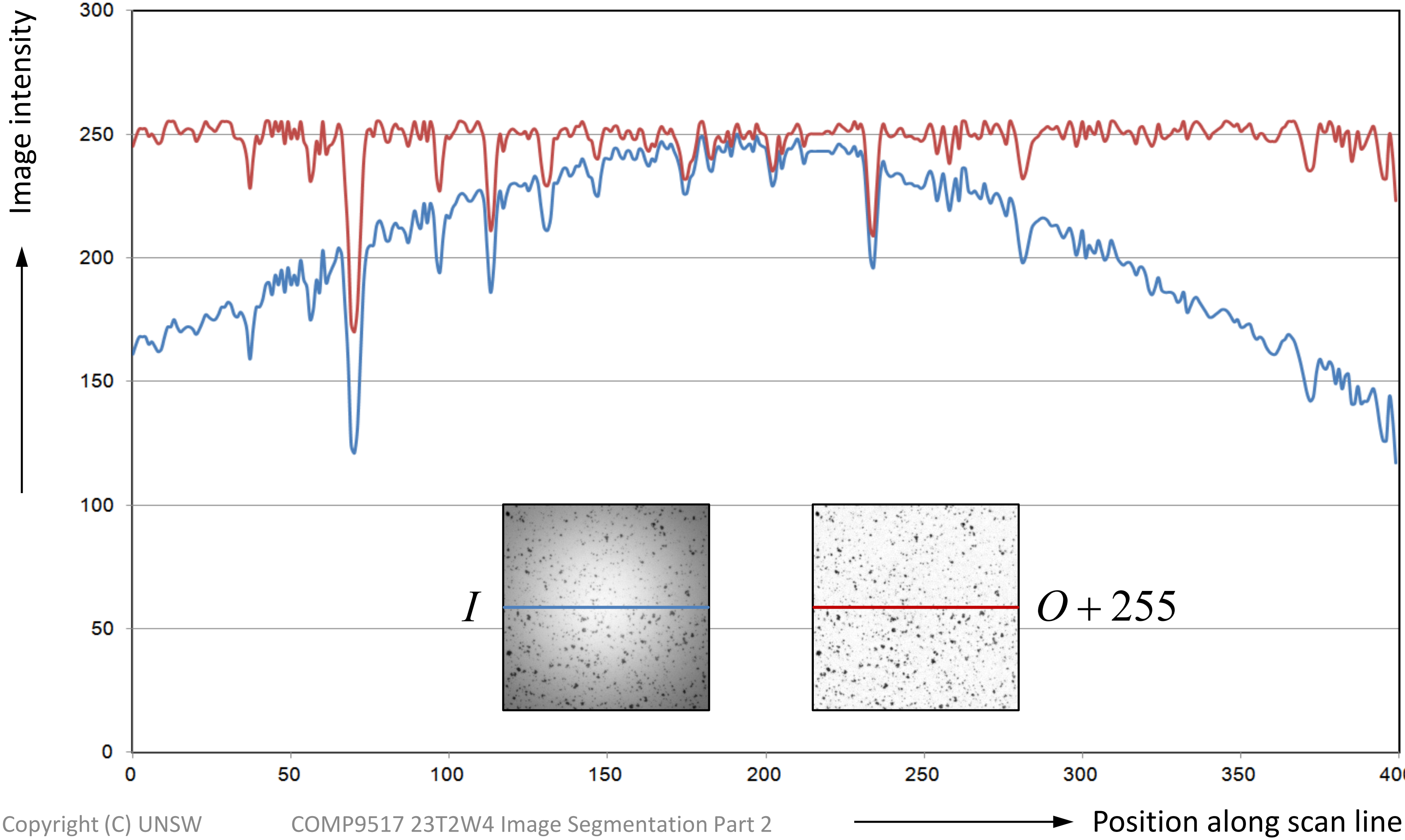
Top-hat filtering of gray-scale images



Top-hat filtering of gray-scale images



Top-hat filtering of gray-scale images



Summary of mathematical morphology

Powerful toolbox of methods around image segmentation

- **Gray-scale morphology for pre-processing**

  Removal of gray-scale noise

  Removal of background shading

  Removal of unwanted image structures

  } Before segmentation

- **Binary morphology for post-processing**

  Closing holes in objects and background

  Finding the inner or outer outline of objects

  Detecting and separating touching objects

  Extracting representative shapes of objects

  Computing distances within and between objects

  } After segmentation