COMP9517
Computer Vision

2023 Term 2 Week 1

Professor Erik Meijering

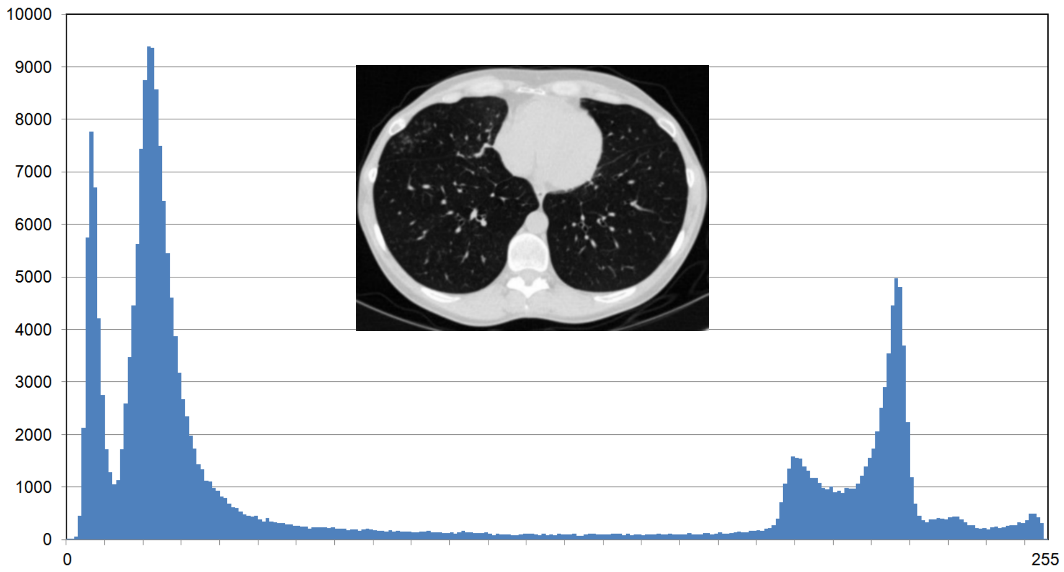


Image Processing

Part 1

What is image processing?

- **Image processing** = image in > image out
- Aims to **suppress distortions** and **enhance relevant information**
- Used to **prepare images for further analysis** and interpretation
- **Image analysis** = image in > features out
- **Computer vision** = image in > interpretation out

Types of image processing

- Two main types of image processing operations:
  - **Spatial domain operations** (in image space)
  - **Transform domain operations** (mainly in Fourier space)

Next week

- Two main types of spatial domain operations:
  - **Point operations** (intensity transformations on individual pixels)
  - **Neighbourhood operations** (spatial filtering on groups of pixels)

Today

Topics and learning goals

- Describe the workings of **basic point operations**

  Contrast stretching, thresholding, inversion, log/power transformations

- Understand and use the **intensity histogram**

  Histogram specification, equalization, matching

- Define **arithmetic and logical operations**

  Summation, subtraction, AND/OR, averaging

Spatial domain operations

- General form of spatial domain operations

$$g(x, y) = T[f(x, y)]$$

where

$f(x, y)$ is the input image

$g(x, y)$ is the processed image

$T[\cdot]$ is the operator applied at (x, y)

Spatial domain operations

- Point operations: T operates on individual pixels

$$T: \mathbb{R} \longrightarrow \mathbb{R} \qquad g(x,y) = T\big(f(x,y)\big)$$

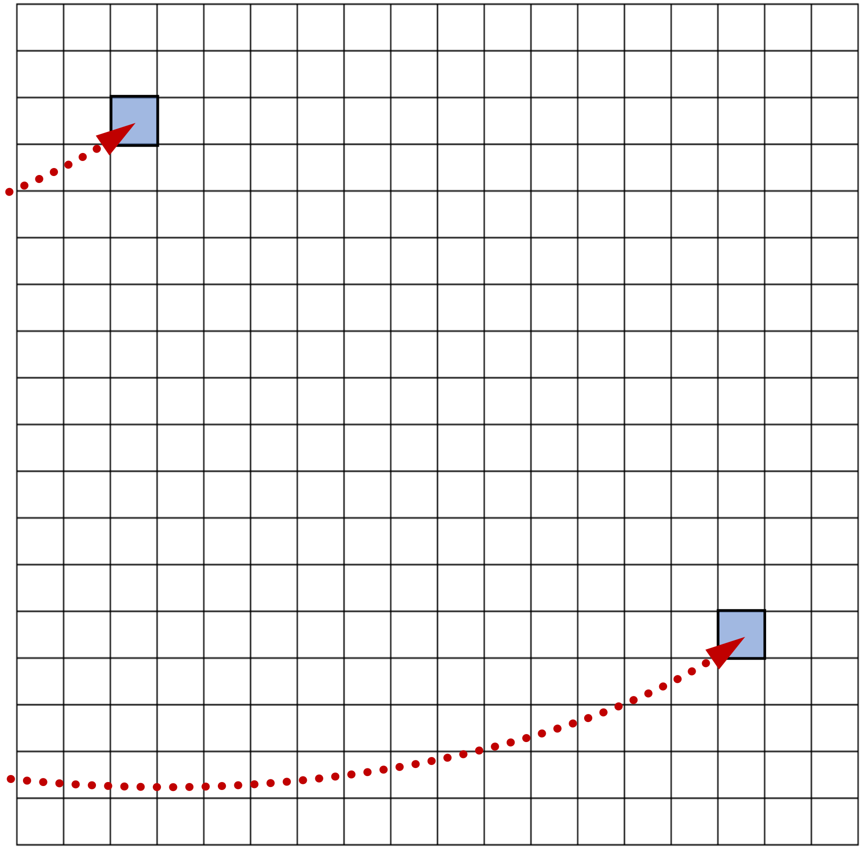
- Neighbourhood operations: T operates on multiple pixels

$$T: \mathbb{R}^2 \longrightarrow \mathbb{R} \qquad g(x,y) = T(f(x,y), f(x+1,y), f(x-1,y), \ldots)$$

Point operations

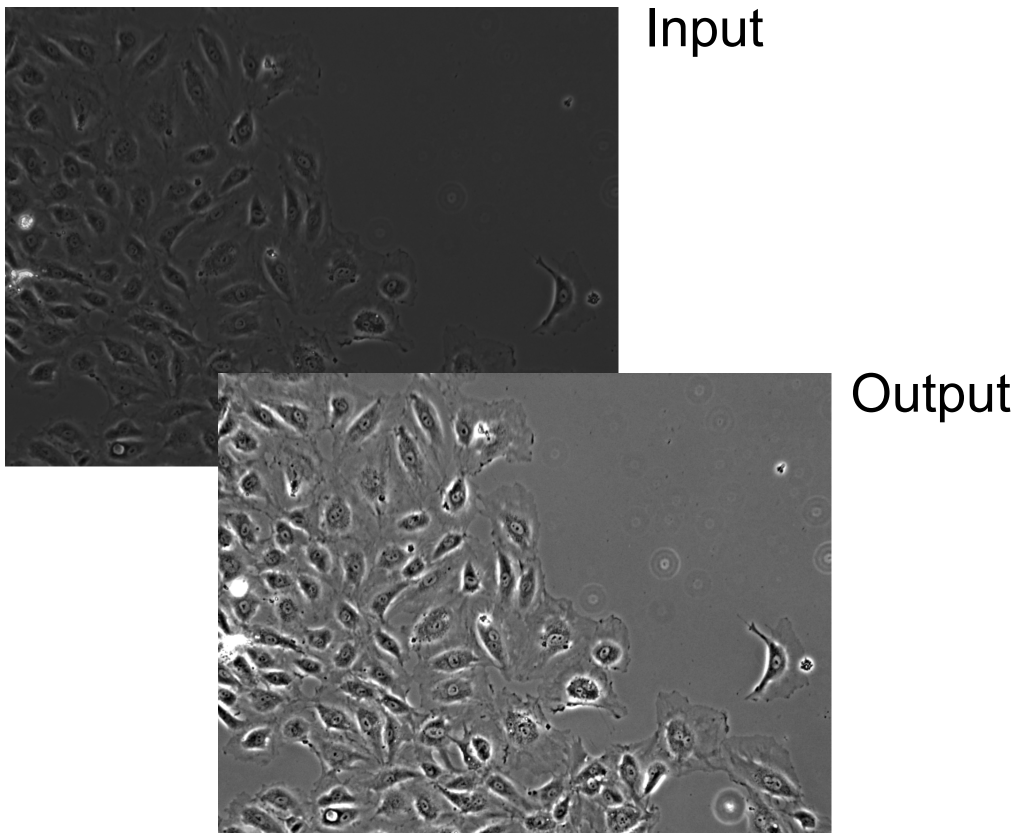
0	2	2	2	5	8	11	8	2	2	0	0	0	0	0	0	0	0
0	0	2	11	76	136	164	85	11	5	2	2	0	0	0	0	0	0
0	2	25	172	181	133	133	164	90	14	5	2	2	0	0	0	0	0
2	5	175	130	104	127	141	164	206	65	31	11	2	2	0	0	0	0
2	28	212	124	110	204	164	232	133	155	218	87	14	2	2	0	0	0
2	73	178	133	121	195	34	31	198	175	204	167	104	14	5	0	0	0
2	45	226	141	113	184	53	59	70	192	133	138	167	99	11	2	0	0
0	2	70	184	102	116	155	161	175	155	141	184	255	138	34	5	2	0
0	0	5	141	121	133	209	215	133	206	124	121	130	153	104	8	2	0
0	0	2	73	164	124	121	198	252	147	121	127	119	119	150	19	2	0
0	0	0	5	93	150	102	119	130	127	104	121	124	133	153	25	2	0
0	0	0	0	5	62	153	155	119	136	198	155	127	124	187	19	2	2
0	0	0	0	0	5	138	161	178	155	127	124	141	158	232	5	2	2
0	0	0	0	0	0	11	113	164	172	184	102	121	164	79	2	2	2
0	0	0	0	0	0	2	5	36	206	187	147	164	153	5	2	2	0
0	0	0	0	0	0	0	0	2	5	25	76	31	2	2	2	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

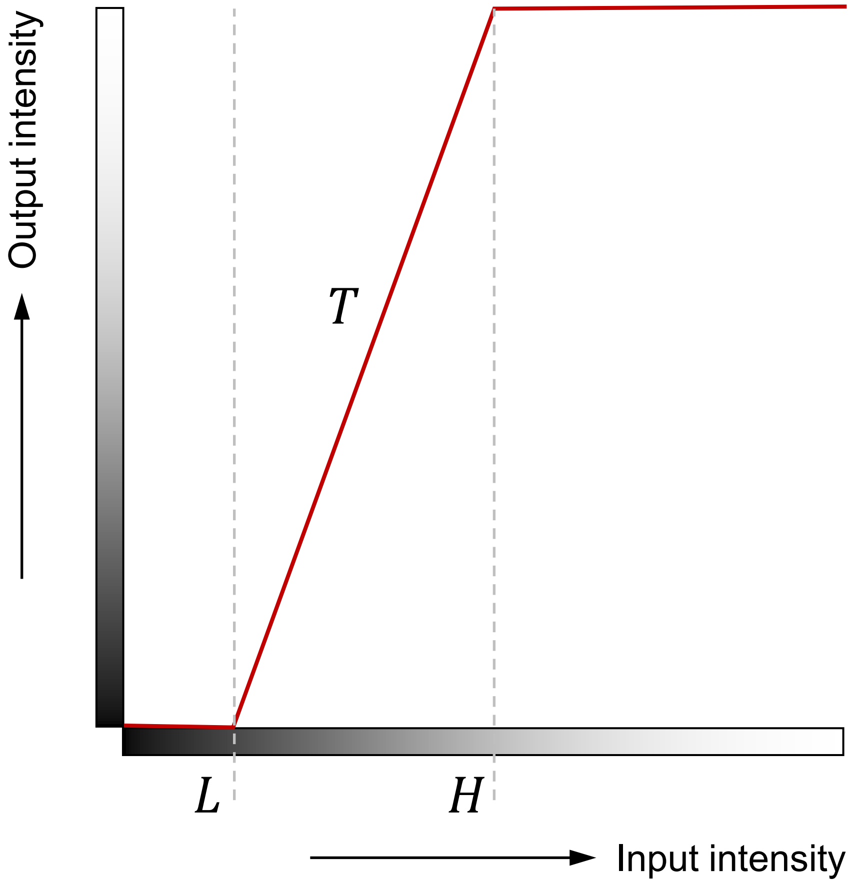
Input image



Output image

Contrast stretching

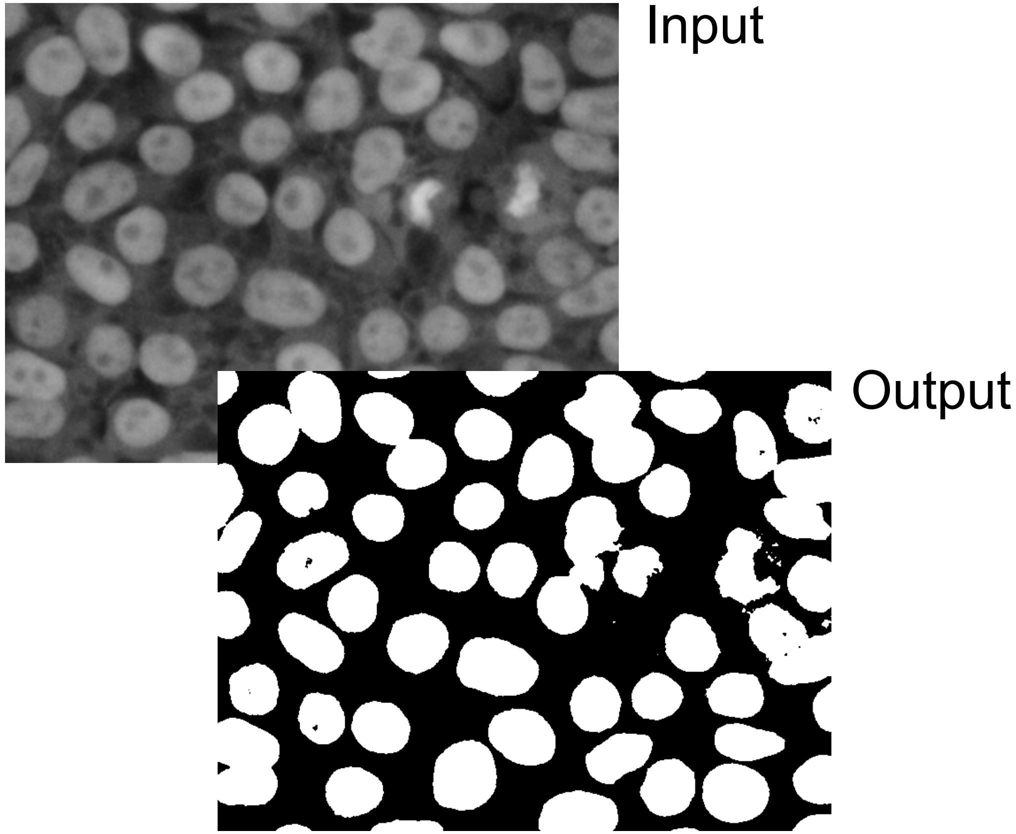


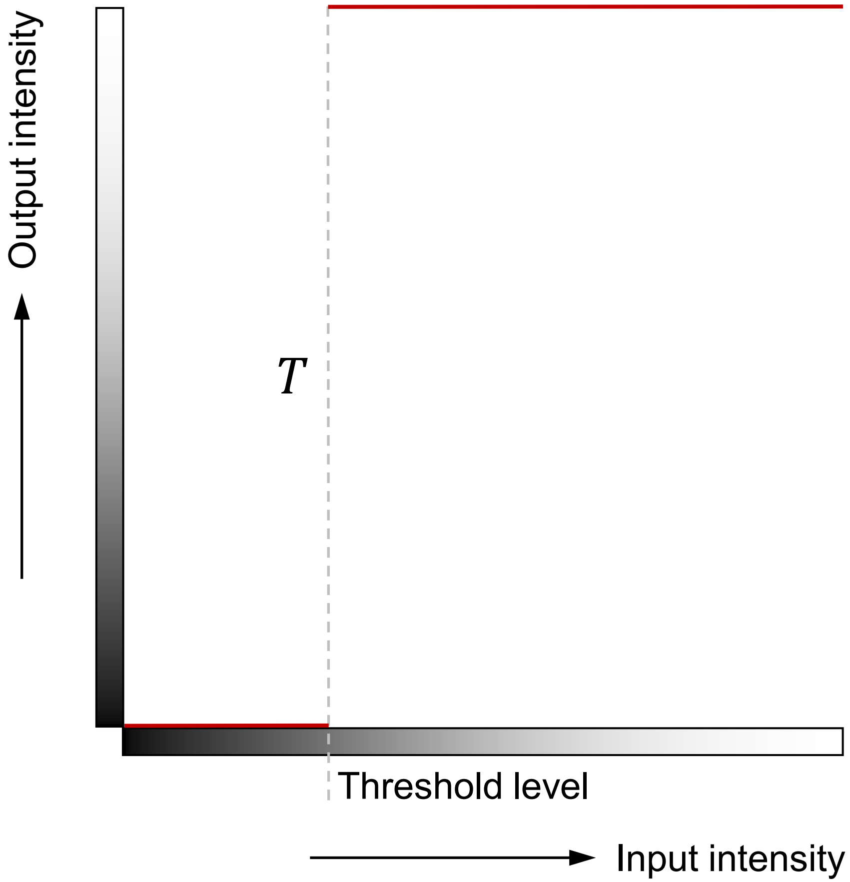


Contrast stretching

- Produces images of higher contrast
- Puts values below L in the input to the minimum (black) in the output
- Puts values above H in the input to the maximum (white) in the output
- Linearly scales values between L and H (inclusive) in the input to between the minimum (black) and the maximum (white) in the output

Intensity thresholding

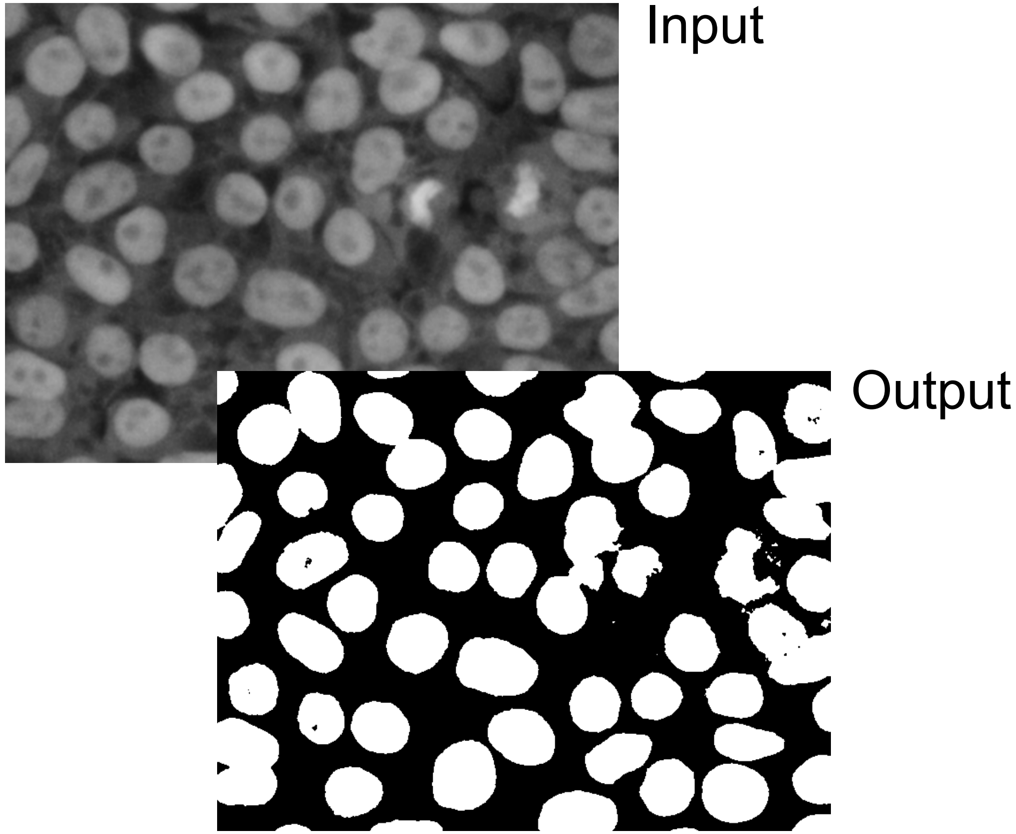


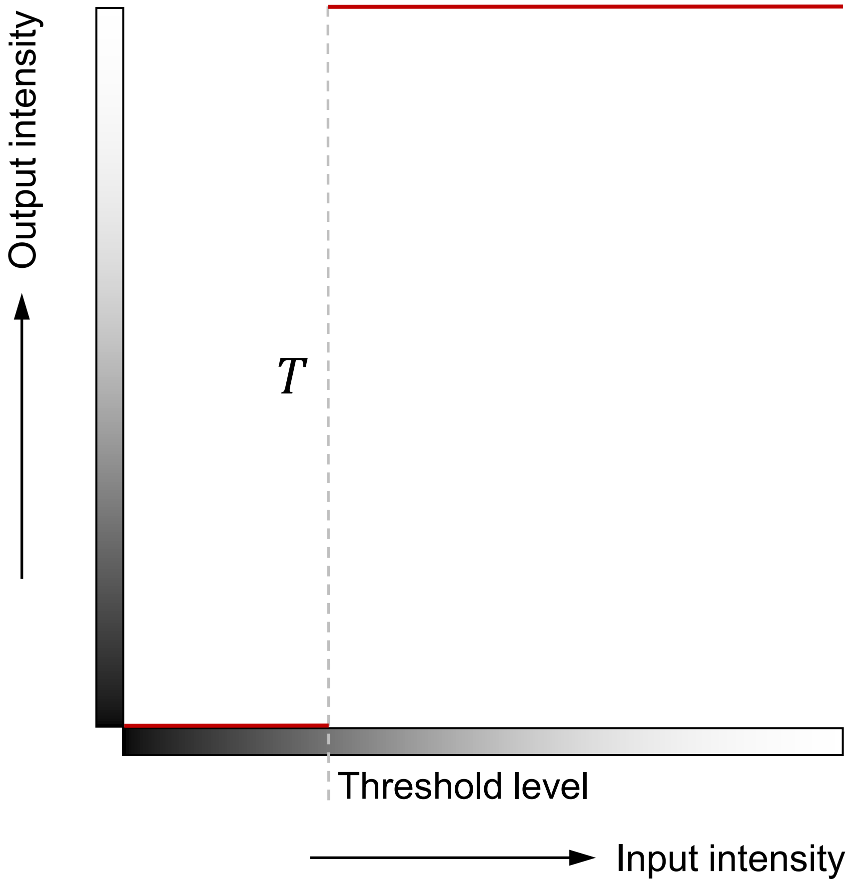


Intensity thresholding

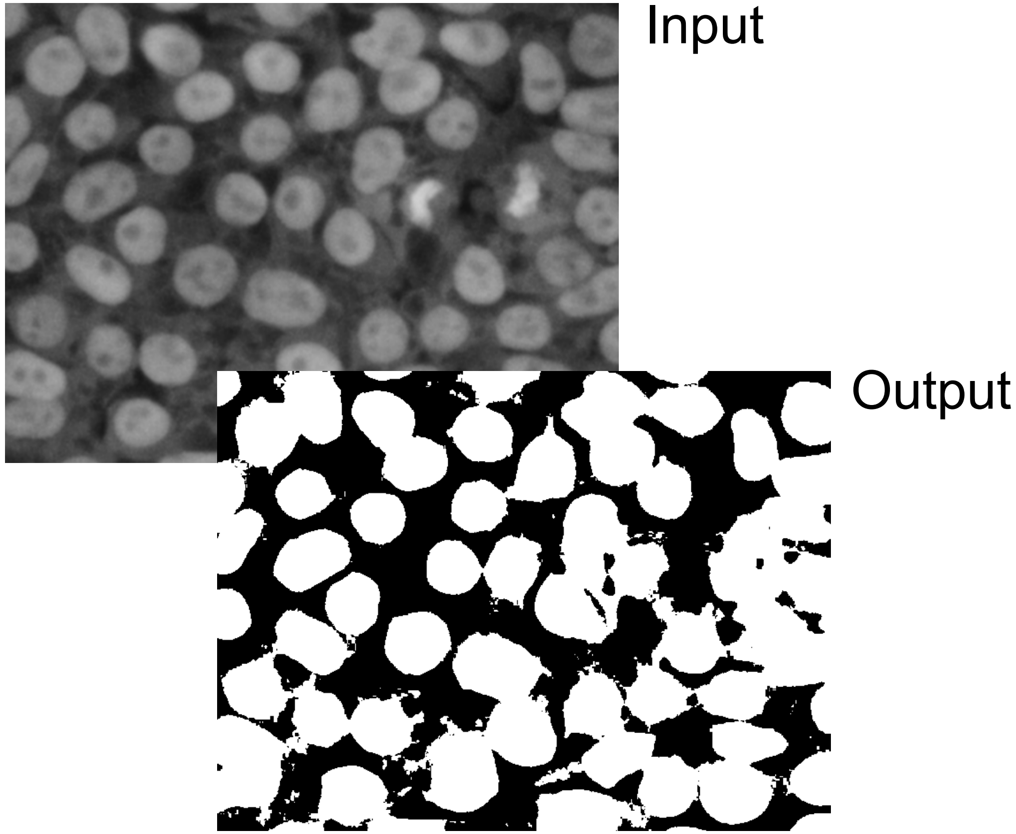
- Limiting case of contrast stretching
- Produces binary images of gray-scale images
- Puts values below the threshold to black in the output
- Puts values equal/above the threshold to white in the output
- Popular method for image segmentation (discussed later)
- Useful only if object and background intensities are very different
- Result depends strongly on the threshold level (user parameter)

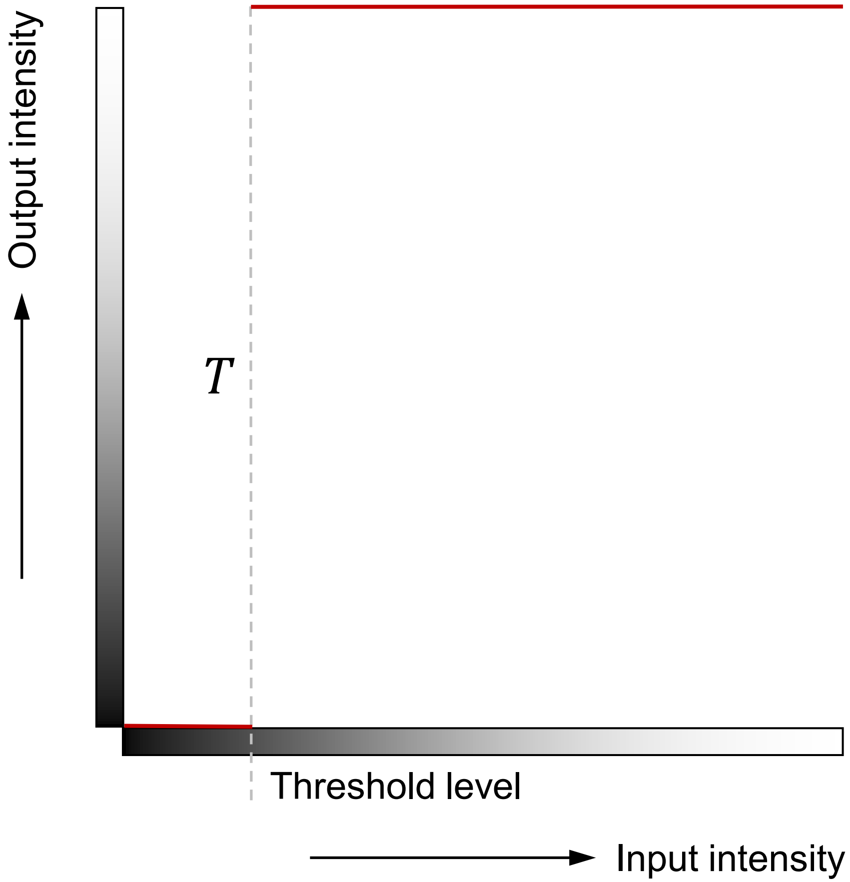
Intensity thresholding



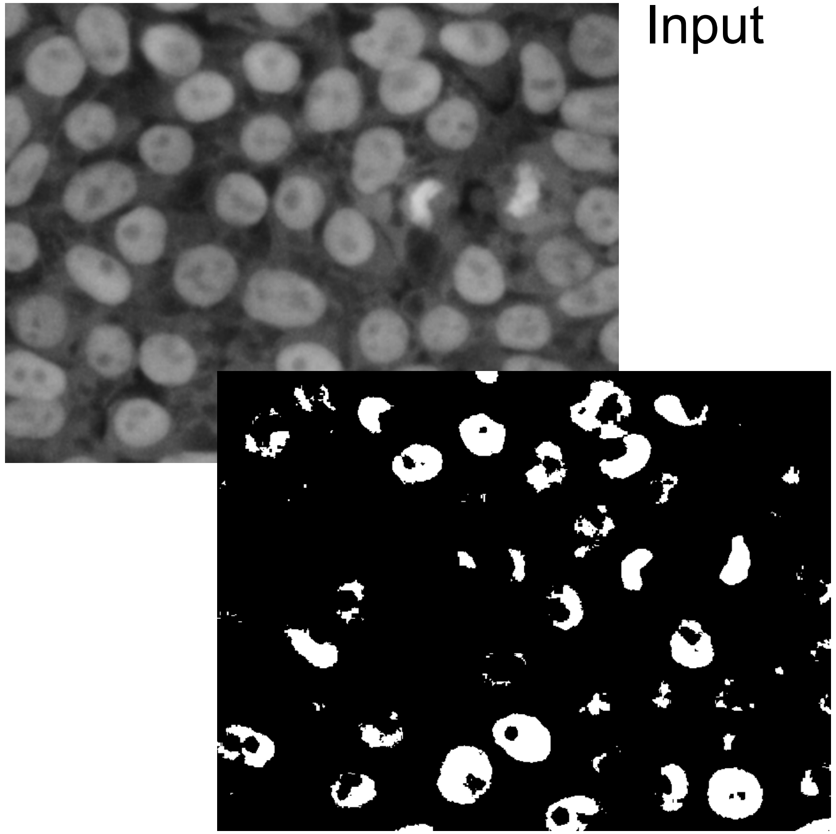


Intensity thresholding



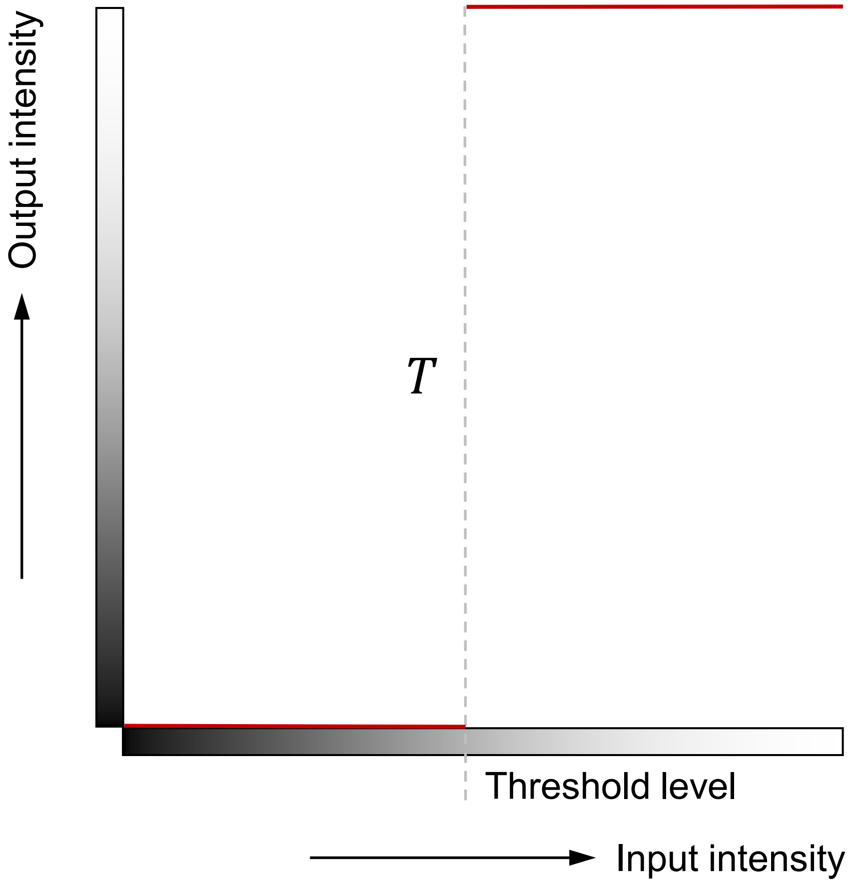


Intensity thresholding



Input

Output



Automatic intensity thresholding

- Otsu's method for computing the threshold automatically https://doi.org/10.1109/TSMC.1979.4310076

Exhaustively searches for the threshold **minimising the intra-class variance**

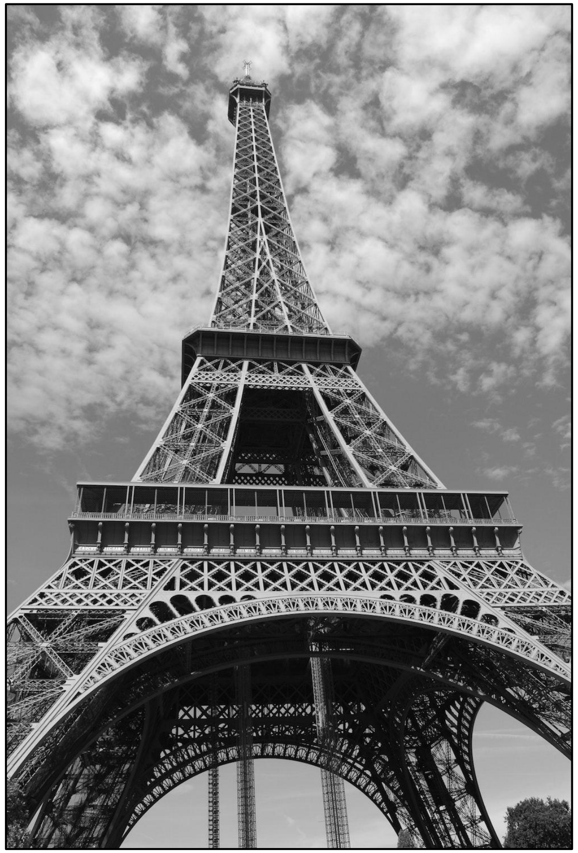
$$\sigma_W^2 = p_0\sigma_0^2 + p_1\sigma_1^2$$

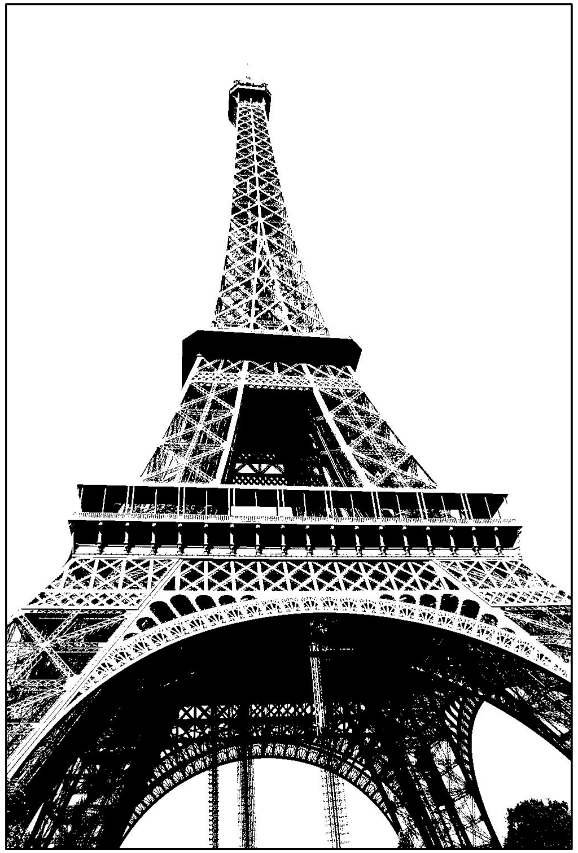
Equivalent to **maximising the inter-class variance** (much faster to compute)

$$\sigma_B^2 = p_0 p_1(\mu_0 - \mu_1)^2$$

Here, p_0 is the fraction of pixels below the threshold (class 0), p_1 is the fraction of pixels equal to or above the threshold (class 1), μ_0 and μ_1 are the mean intensities of pixels in class 0 and class 1, σ_0^2 and σ_1^2 are the intensity variances, and $p_0 + p_1 = 1$ and $\sigma_0^2 + \sigma_1^2 = \sigma^2$

Otsu thresholding example





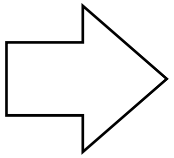
Automatic intensity thresholding

- Isodata method for computing the threshold automatically

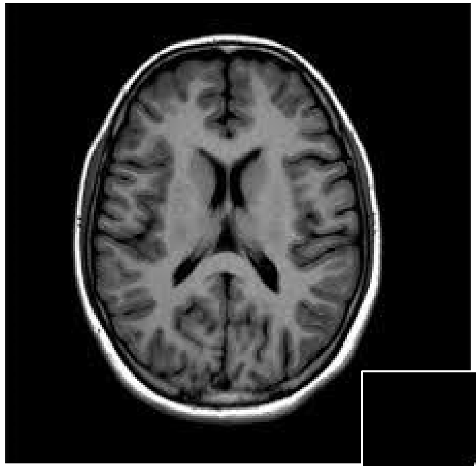
1. Select an arbitrary initial threshold t
2. Compute μ_0 and μ_1 with respect to the threshold
3. Update the threshold to the mean of the means: $t = (\mu_0 + \mu_1)/2$
4. If the threshold changed in Step 3, go to Step 2

Upon convergence, the threshold is midway between the two class means

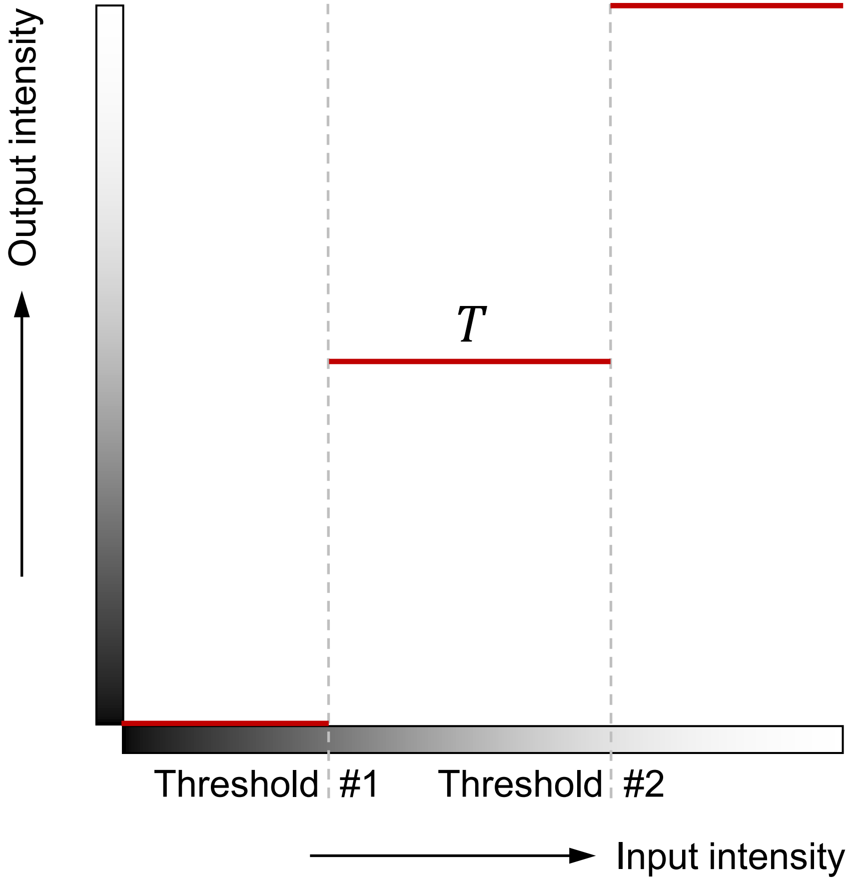
Isodata thresholding example



Multilevel thresholding


Input

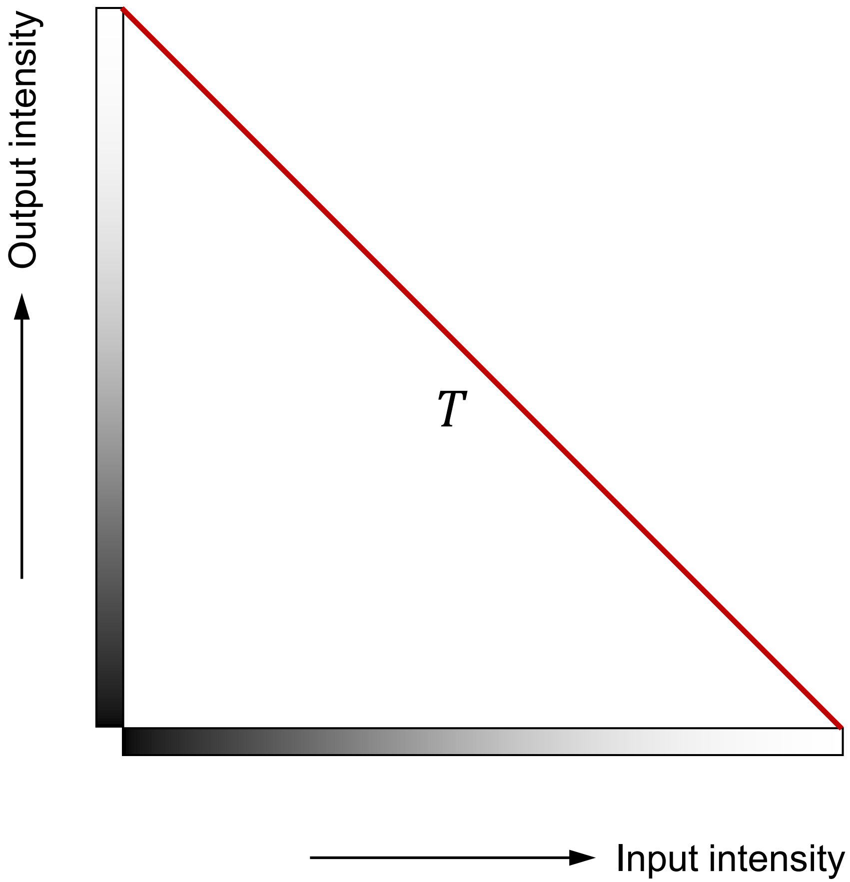

Output



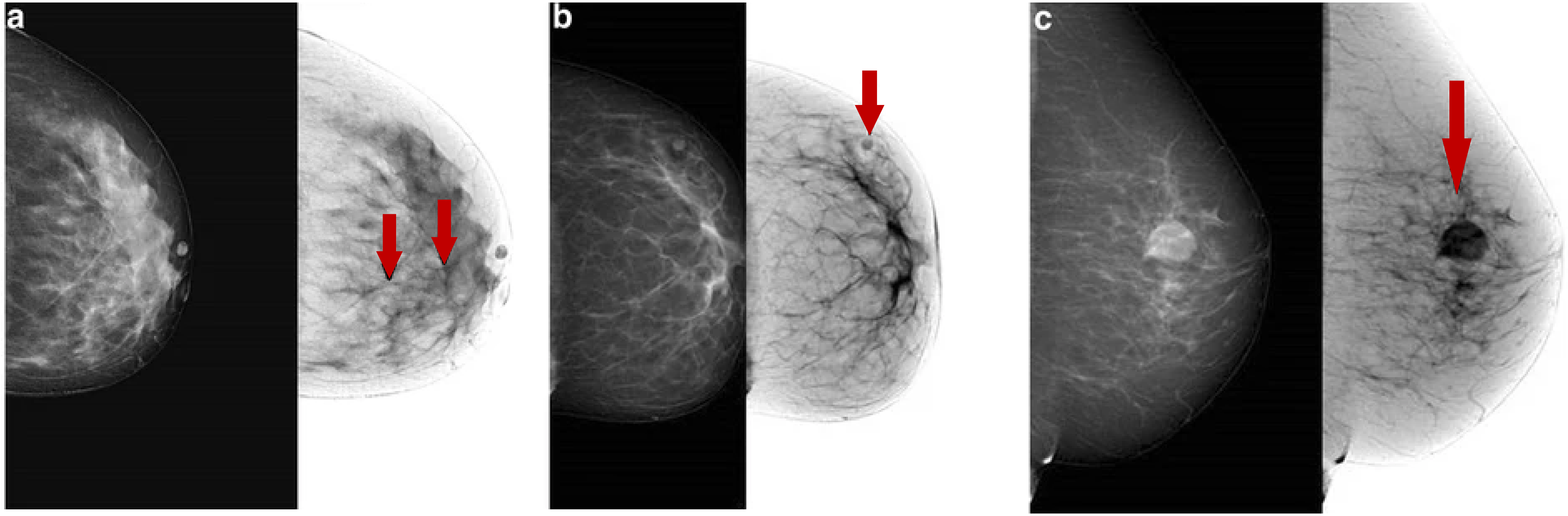
Intensity inversion

Input

Output



Intensity inversion examples



"Assessment of grayscale inverted images in addition to standard images facilitates the detection of microcalcification." https://doi.org/10.1186/s12880-017-0196-6

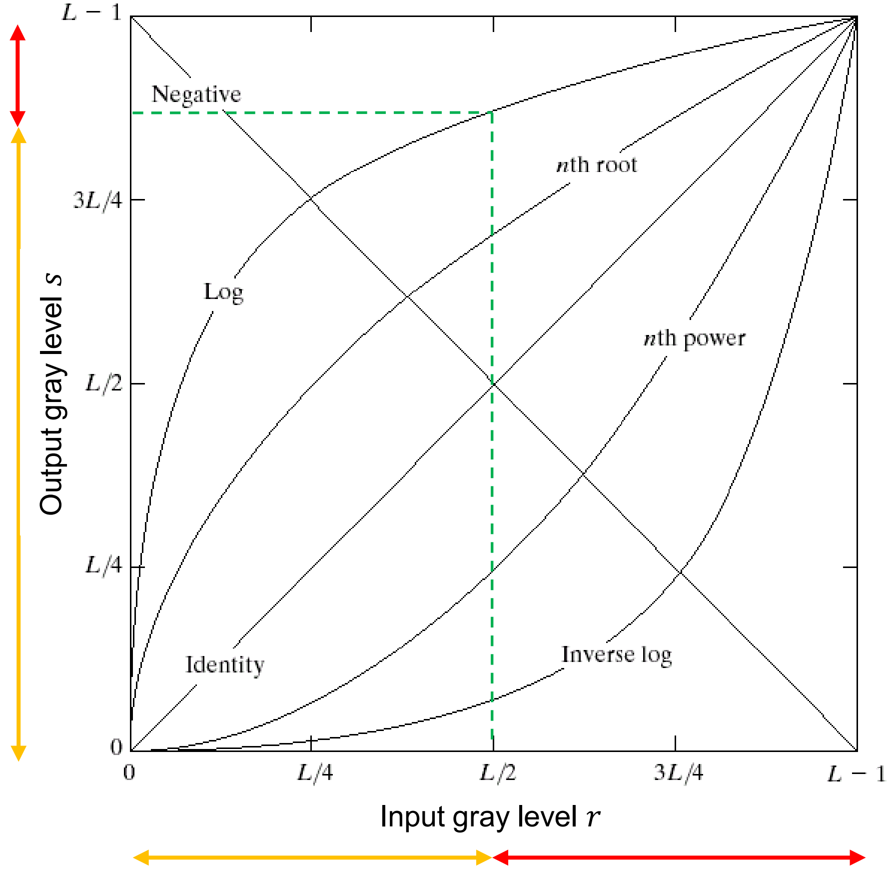
Log transformation

- Definition of log transformation

$$s = c\log(1 + r)$$

where r is the input intensity, s is the output intensity, and c is a constant

  - Maps a narrow input range of low gray-level values into a wider range of output values, and vice versa for higher gray-level values
  - Also compresses the dynamic range of images with large variations in pixel values (such as Fourier spectra, to be discussed later)



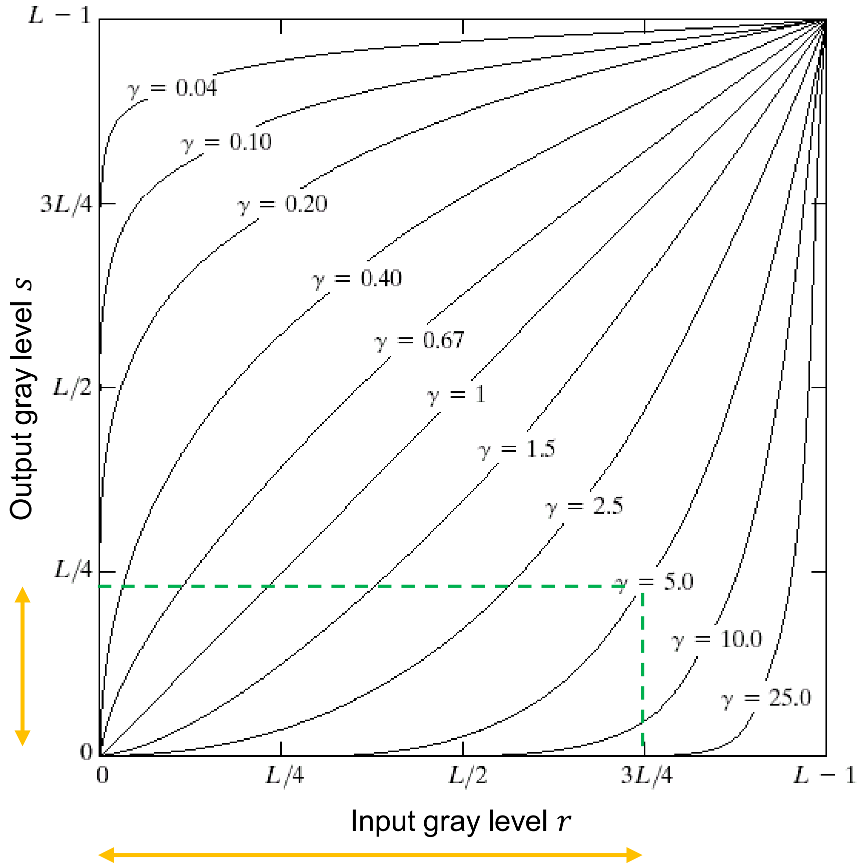
Power transformation

- Definition of power transformation

$$s = c\, r^{\gamma}$$

where c and γ are constants

  - Similar to log transformation
  - Represents a family of transformations by varying γ
  - Many devices respond according to a power law
  - Example power transformation: gamma correction
  - Useful for general-purpose contrast manipulation



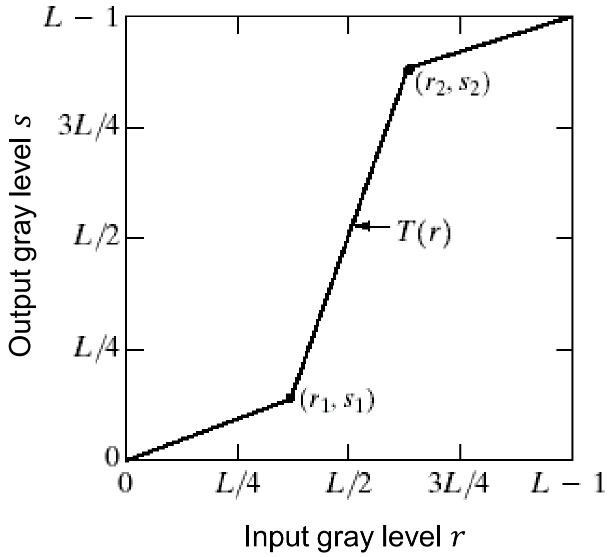
Power transformation examples

$c = 1$

Input | $\gamma = 3$ | $\gamma = 4$ | $\gamma = 5$

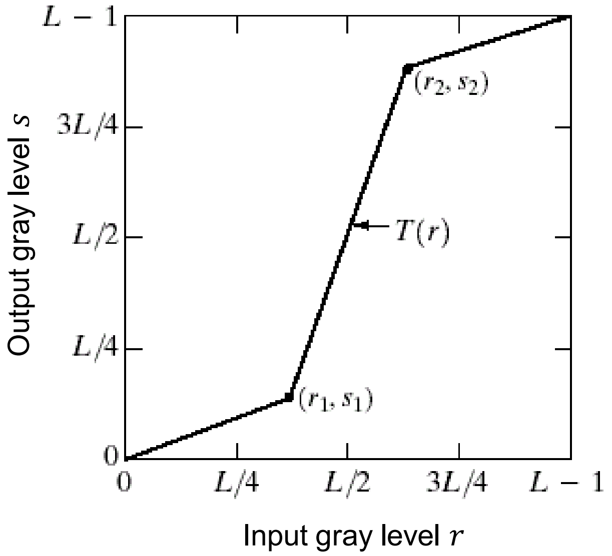
Piecewise linear transformations

- Complementary to other transformation methods
- Enable more fine-tuned design of transformations
- Can have very complex shapes
- Requires more user input



Piecewise contrast stretching

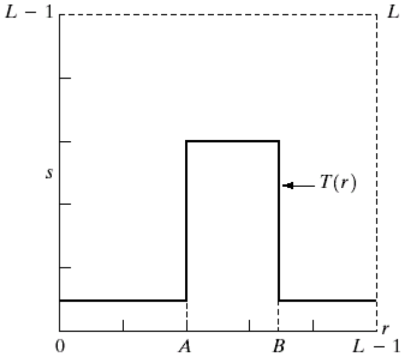
- One of the simplest piecewise linear transformations
- Increases the dynamic range of gray levels in images
- Used in display devices or recording media to span full range



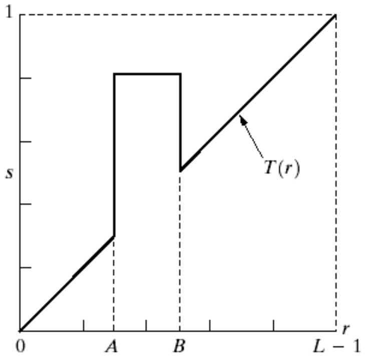
Gray-level slicing

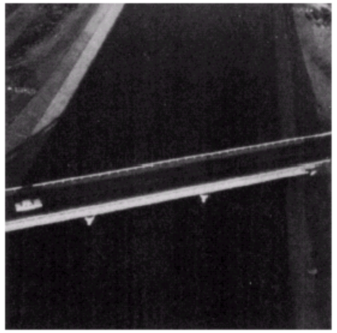
- Used to highlight a specific range of gray levels
- Two different slicing approaches:
  1) High value for all gray levels in a range of interest and low value for all others (produces a binary image)
  2) Brighten a desired range of gray levels while preserving background and other gray-scale tones of the image

Transform 1

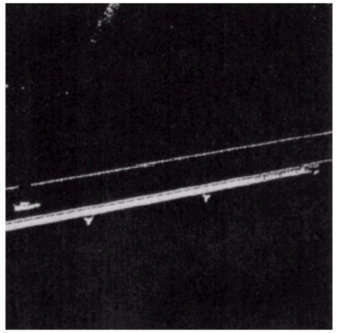


Transform 2





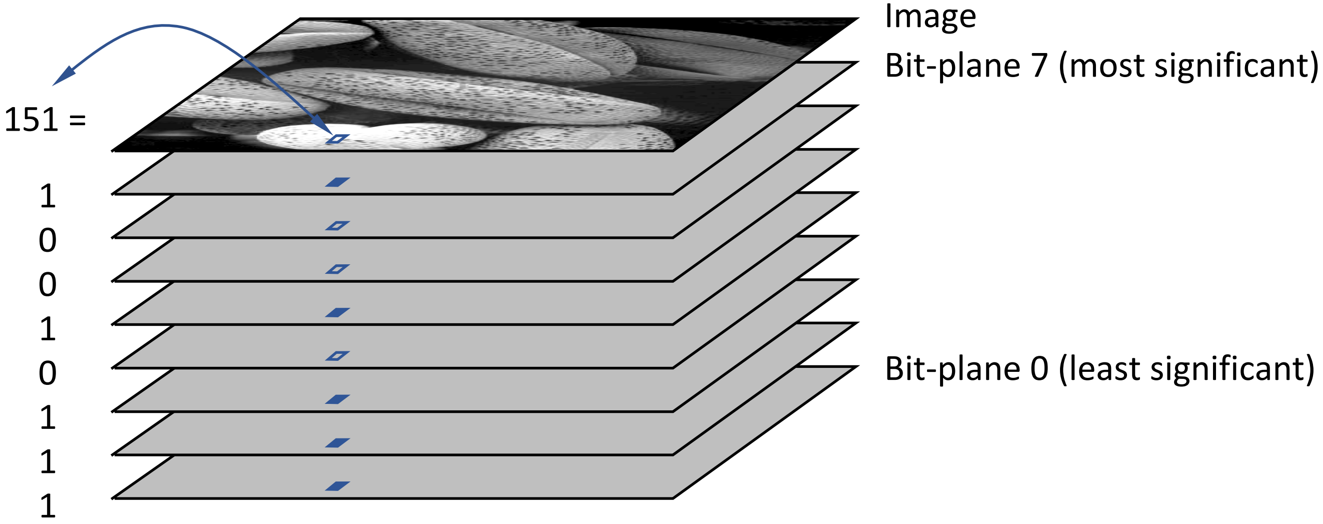
Input



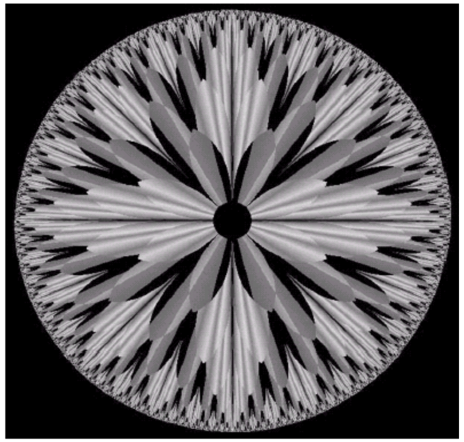
Result of 1

Bit-plane slicing

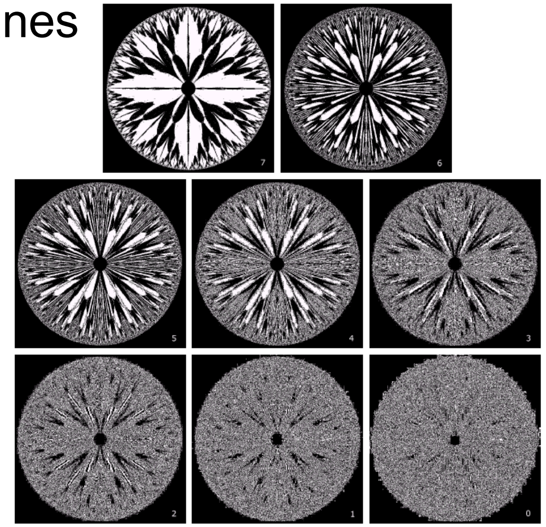
- Highlights contribution to total image by specific bits
- An image with *n* bits/pixel has *n* bit-planes
- Can be useful for image compression



Input

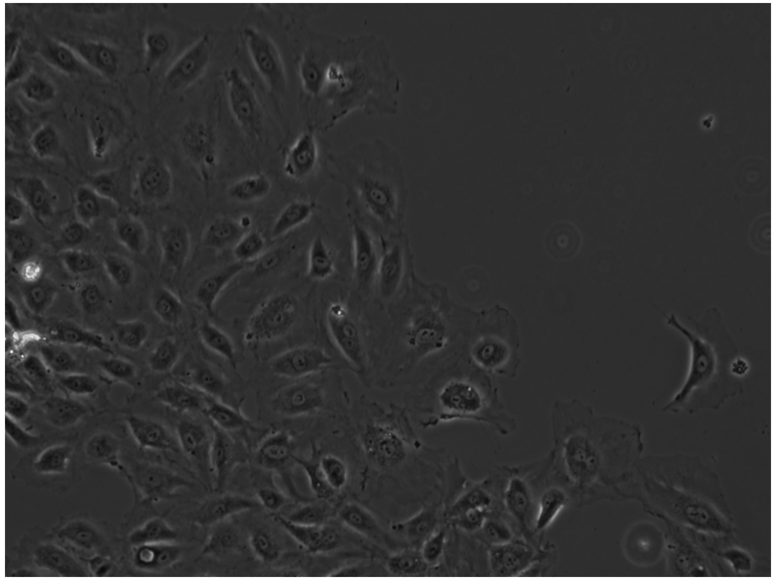


Bit-planes

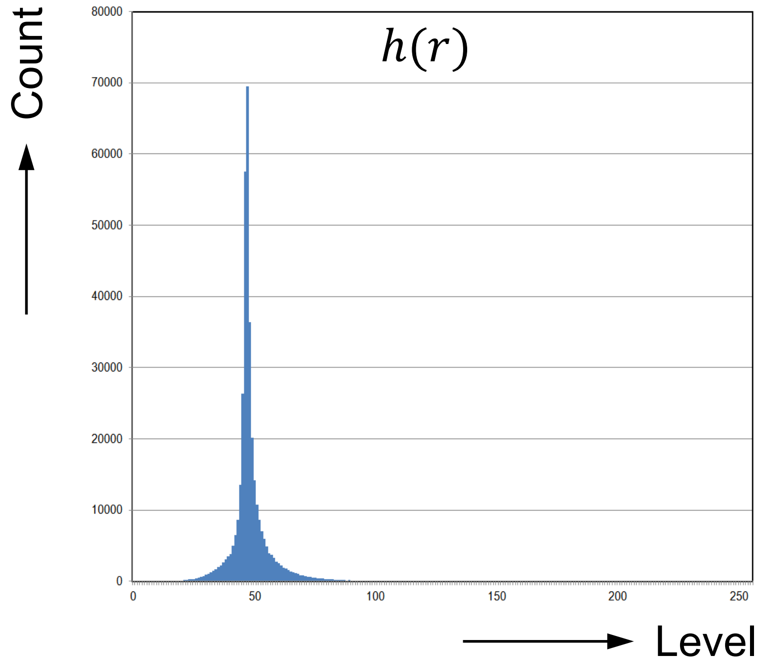


Histogram of pixel values

- For every possible gray-level value, count the number of pixels having that value, and plot the pixel counts as a function of gray level



8-bit image



$$L = 2^8 = 256$$

$$N = \#\text{pixels}$$

$$\sum_{r=0}^{L-1} h(r) = N$$

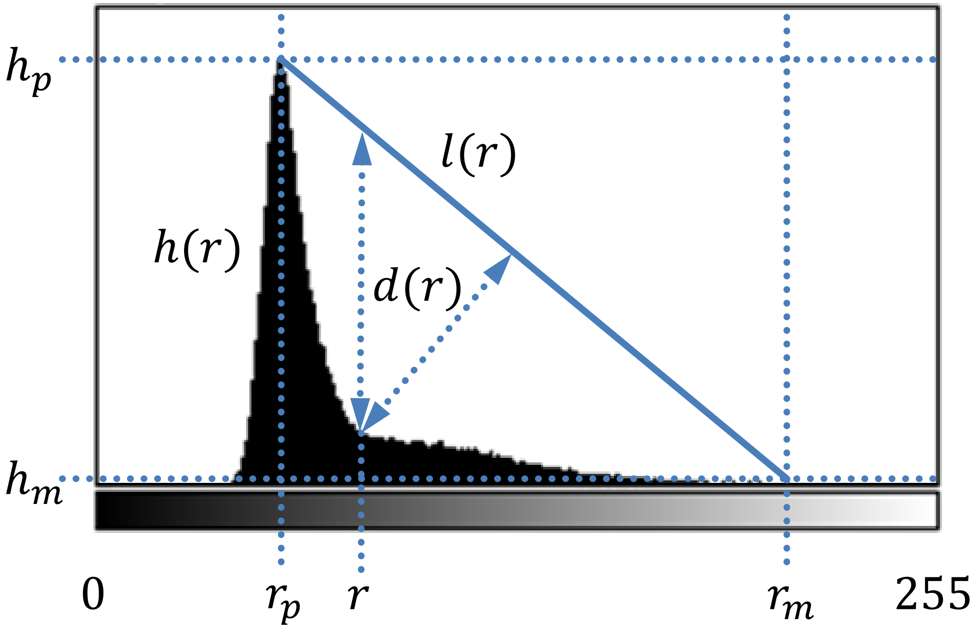
Normalized histogram
= probability function

$$\frac{1}{N} h(r) = p(r)$$

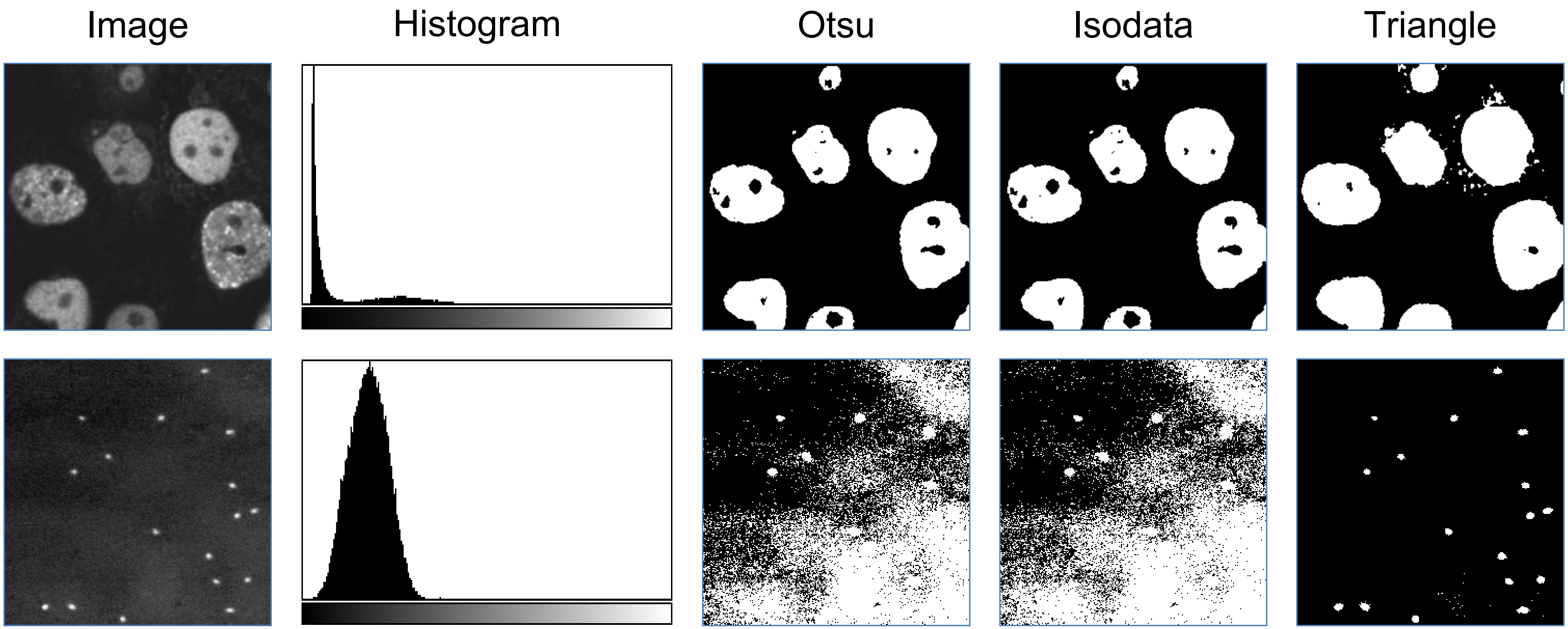
Histogram based thresholding

- Triangle method for computing the threshold automatically

  1. Find the histogram peak (r_p, h_p) and the highest gray level point (r_m, h_m)
  2. Construct a straight line $l(r)$ from the peak to the highest gray level point
  3. Find the gray level r for which the distance $||l(r) - h(r)||$ is the largest



Comparison of thresholding methods



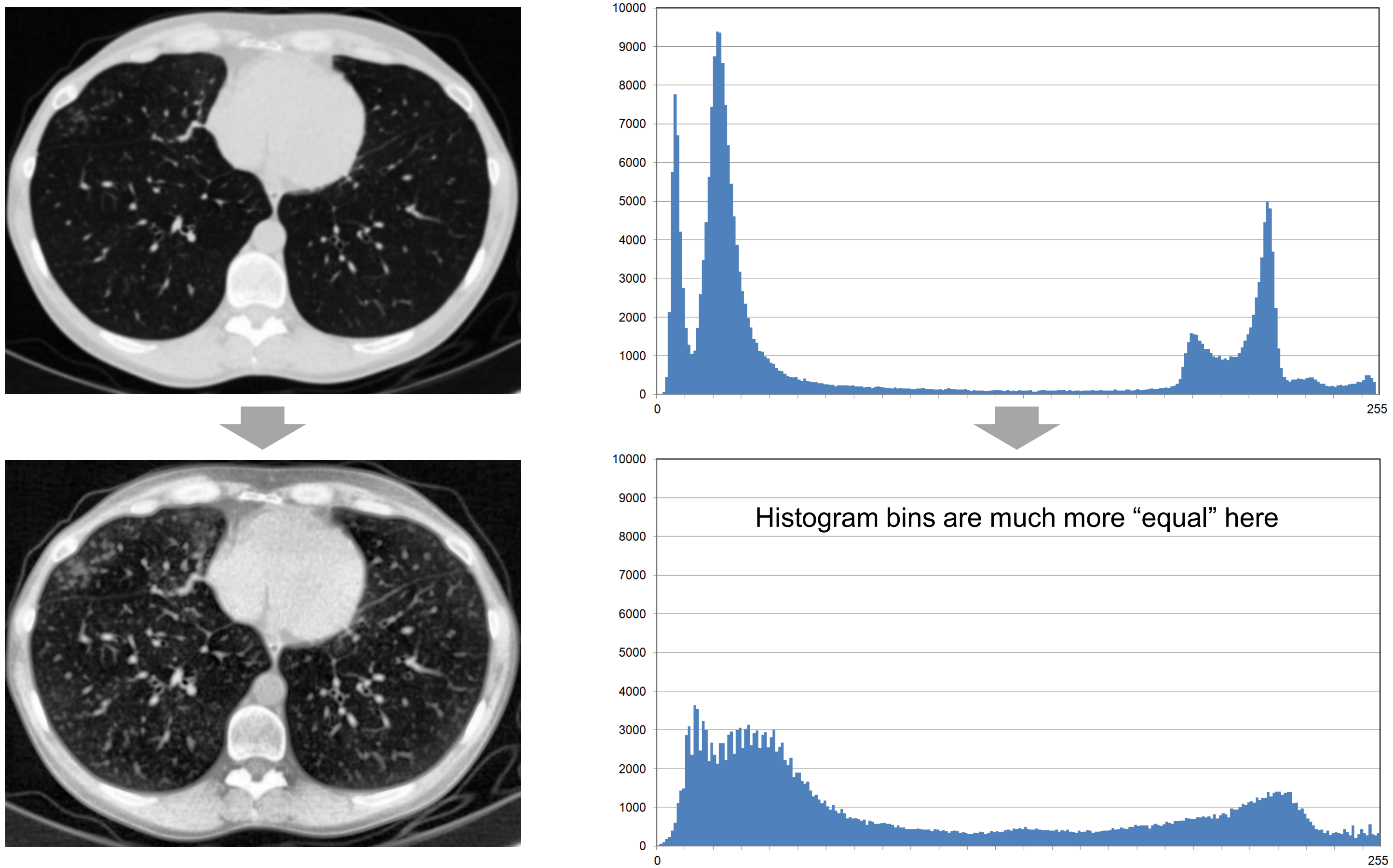
Histogram processing

- **Histogram equalization**
  Aim: To get an image with equally distributed intensity levels over the full intensity range

- **Histogram specification** (also called **histogram matching**)
  Aim: To get an image with a specified intensity distribution, determined by the shape of the histogram

Histogram equalization

Enhances contrast for intensity values near histogram maxima and decreases contrast near histogram minima



Histogram equalization

- Let $r \in [0, L-1]$ represent pixel values (intensities, gray levels)
  $r = 0$ represents black and $r = L - 1$ represents white

- Consider transformations $s = T(r)$, $0 \leq r \leq L - 1$, satisfying

1) $T(r)$ is single-valued and monotonically increasing in $0 \leq r \leq L - 1$
   This guarantees that the inverse transformation $T^{-1}(s)$ exists

2) $0 \leq T(r) \leq L - 1$ for $0 \leq r \leq L - 1$
   This guarantees that the input and output ranges will be the same

Histogram equalization (continuous case)

Consider r and s as continuous random variables over $[0, L-1]$ with PDFs $p_r(r)$ and $p_s(s)$

If $p_r(r)$ and $T(r)$ are known and $T^{-1}(s)$ satisfies monotonicity, then, from probability theory

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

https://www.cl.cam.ac.uk/teaching/2003/Probability/prob11.pdf

Let us choose: $s = T(r) = (L-1)\int_0^r p_r(\xi)d\xi$

This is the CDF (cumulative distribution function) of r which satisfies conditions (1) and (2)

Now: $\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(\xi)d\xi\right] = (L-1)p_r(r)$

Therefore: $p_s(s) = p_r(r)\left|\frac{1}{(L-1)p_r(r)}\right| = \frac{1}{L-1}$ for $0 \leq s \leq L-1$ (uniform distribution)

Histogram equalization (discrete case)

For discrete values we get probabilities and summations instead of PDFs and integrals:

$$p_r(r_k) = n_k / MN \text{ for } k = 0, 1, \dots, L-1$$

where MN is total number of pixels in image, n_k is the number of pixels with gray level r_k and L is the total number of gray levels in the image

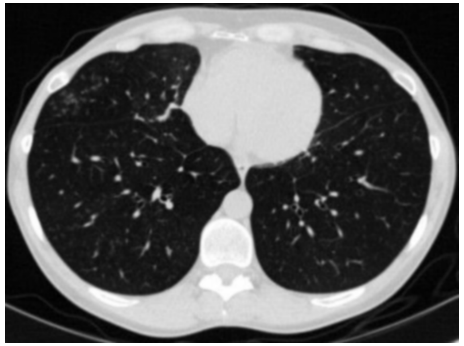
Thus: $s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{L-1}{MN}\sum_{j=0}^{k} n_j$ for $k = 0, 1, \dots, L-1$

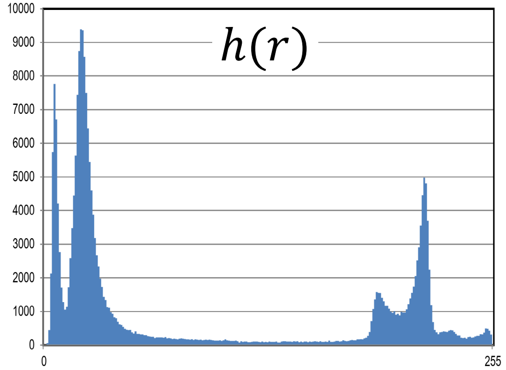
This transformation is called *histogram equalization*

However, in practice, getting a perfectly uniform distribution for discrete images is rare

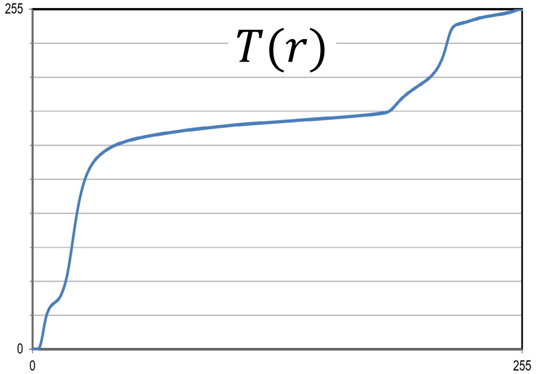
Constrained histogram equalization

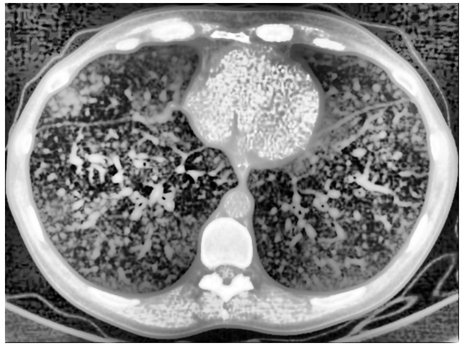
Input

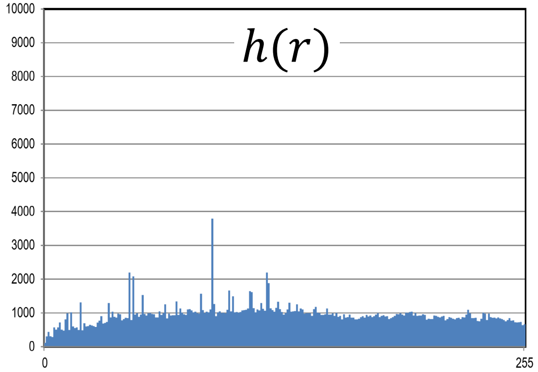


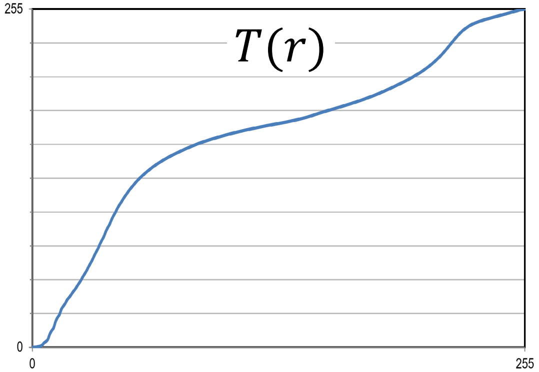


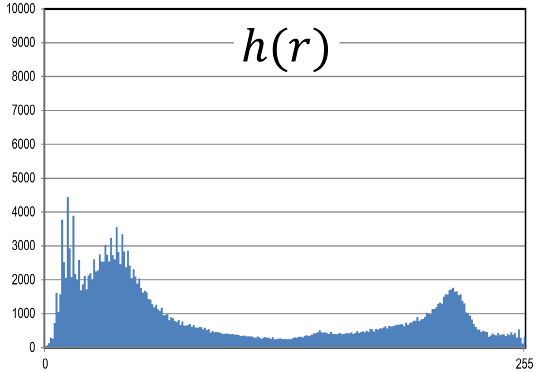
Full histogram equalization (slope of $T(r)$ is unconstrained)











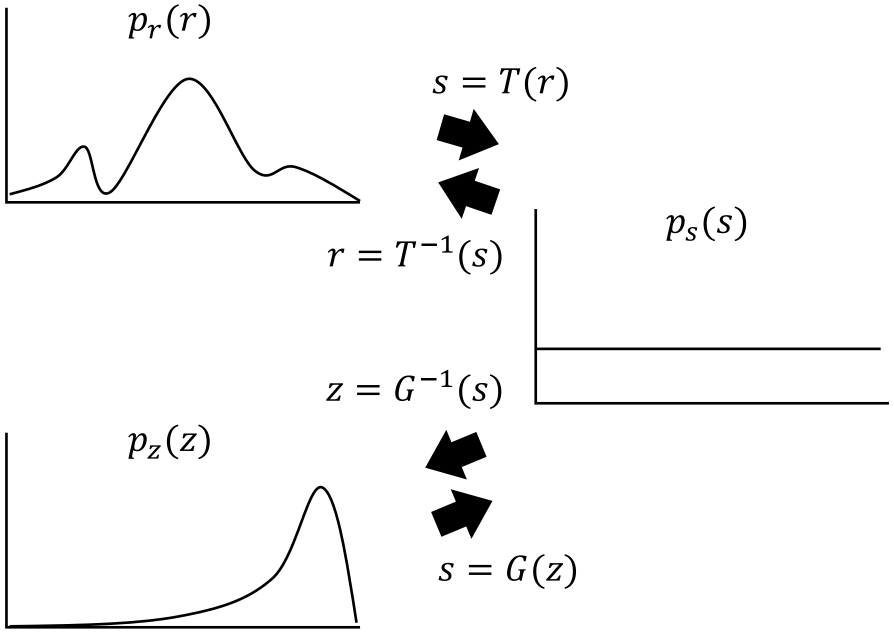
Constrained histogram equalization (slope of $T(r)$ is constrained)

Histogram matching (continuous case)

Assume that r and s are continuous and $p_z(z)$ is the target distribution for the output image

From our previous analysis we know the following transformation results in a uniform distribution:

$$s = T(r) = (L-1)\int_0^r p_r(\xi)d\xi$$



Now we can define a function $G(z)$ as:

$$G(z) = (L-1)\int_0^z p_z(\xi)d\xi = s$$

Therefore:

$$z = G^{-1}(s) = G^{-1}[T(r)]$$

Histogram matching (discrete case)

For discrete image values we can write:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{L-1}{MN}\sum_{j=0}^{k} n_j$$

$$k = 0, 1, \ldots, L-1$$

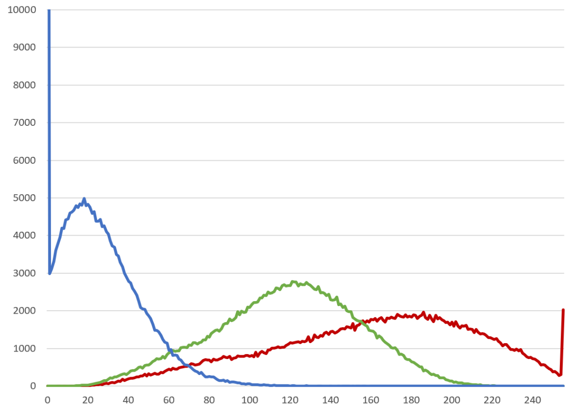
And: $$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

Therefore: $$z_q = G^{-1}(s_k)$$

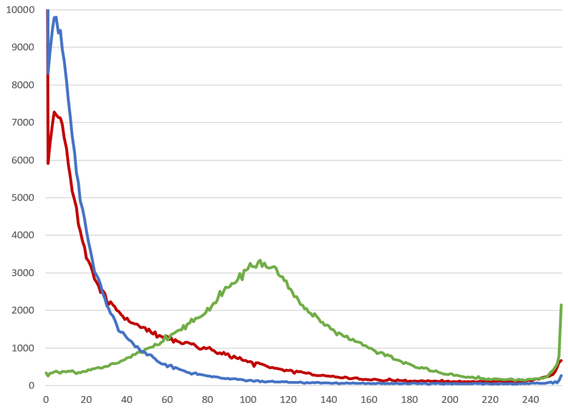
Histogram matching example

https://automaticaddison.com/tag/image-processing/page/3/

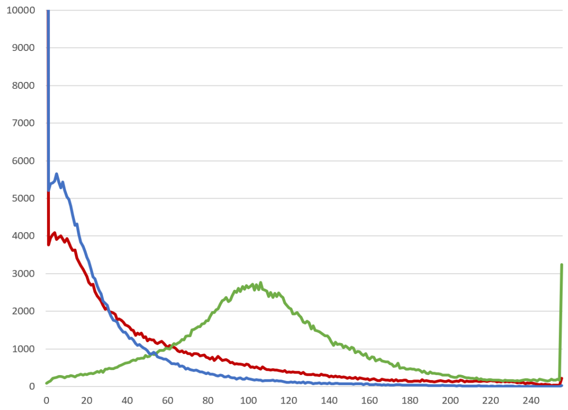
Input



Target



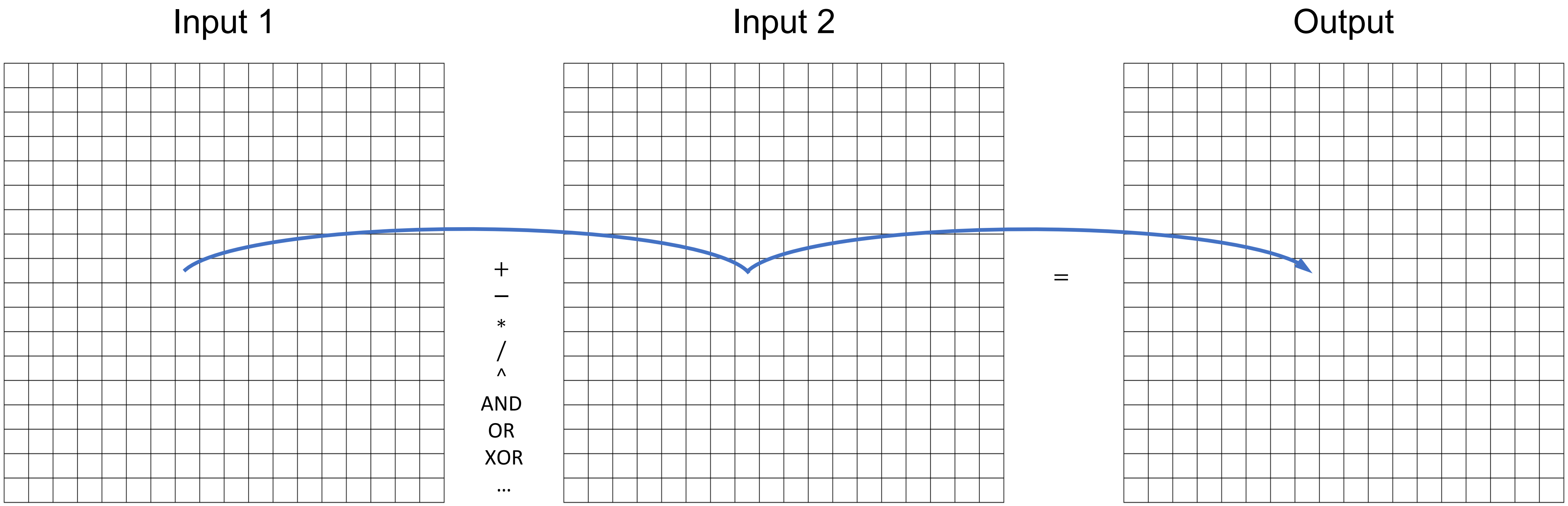
Output



Matching done for each colour channel (R,G,B)

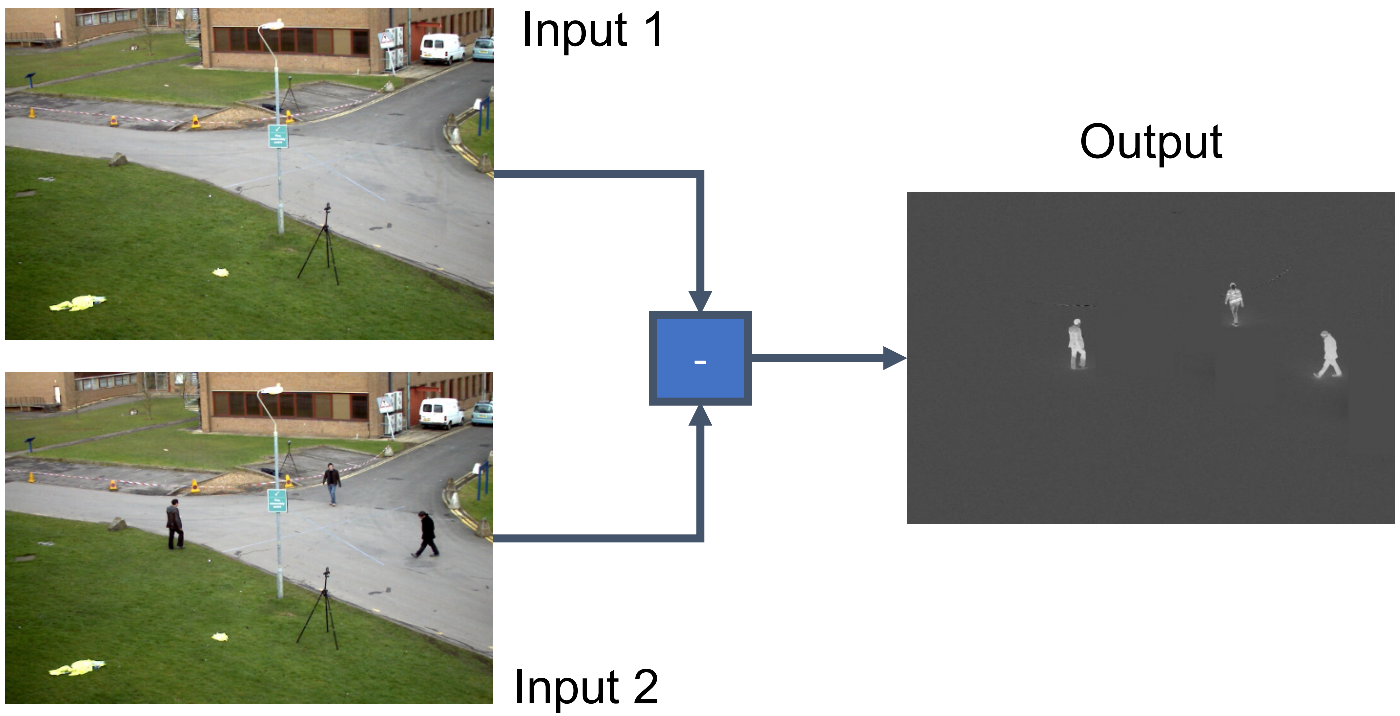
Arithmetic and logical operations

- Defined on a pixel-by-pixel basis between two images



Arithmetic and logical operations

- Useful arithmetic operations include addition and subtraction



Arithmetic and logical operations

- Useful logical operations include bitwise AND and OR

Input

Mask

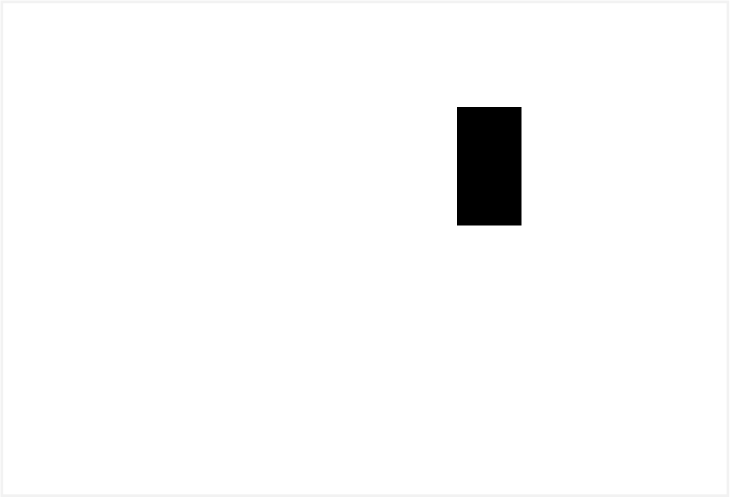
Input AND Mask

Arithmetic and logical operations

- Useful logical operations include bitwise AND and OR

Input

Mask



Input OR Mask

Averaging

- Useful for example to reduce noise in images

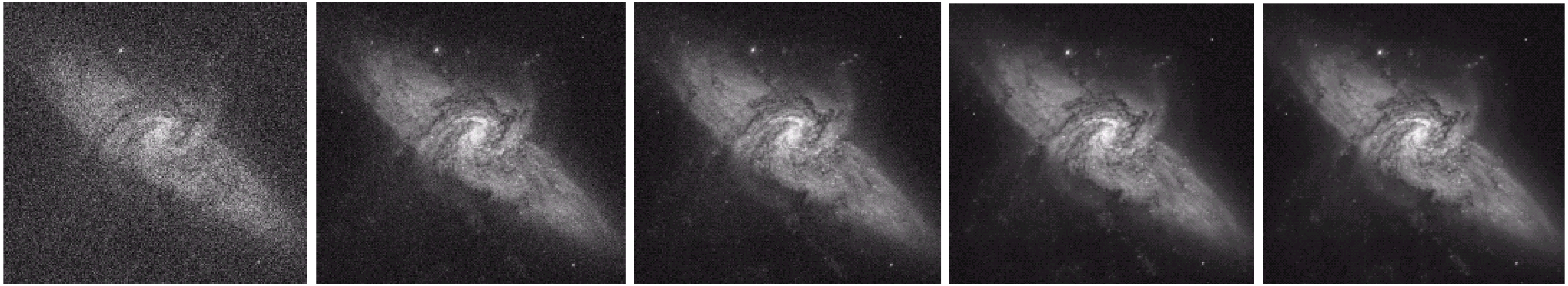
Assume the true noise-free image is $g(x, y)$ and the actual observed images are $f_i(x, y) = g(x, y) + n_i(x, y)$ for $i = 1, \ldots, N$, where the n_i are zero-mean, independent and identically distributed (i.i.d.) noise images, then we have $\mathrm{E}[f_i(x, y)] = g(x, y)$ and $\mathrm{VAR}[f_i(x, y)] = \mathrm{VAR}[n_i(x, y)] = \sigma^2(x, y)$

$$\rightarrow \bar{f}(x, y) = \frac{1}{N}\sum_{i=1}^{N} f_i(x, y) = \frac{1}{N}\sum_{i=1}^{N}[g(x, y) + n_i(x, y)] = g(x, y) + \frac{1}{N}\sum_{i=1}^{N} n_i(x, y)$$

$$\rightarrow \mathrm{VAR}\left[\frac{1}{N}\sum_{i=1}^{N} n_i(x, y)\right] = \frac{1}{N^2}\sum_{i=1}^{N}\mathrm{VAR}[n_i(x, y)] = \frac{1}{N^2} N\sigma^2(x, y) = \frac{\sigma^2(x, y)}{N}$$

Averaging

- Useful for example to reduce noise in images



$N = 1$	$N = 8$	$N = 16$	$N = 64$	$N = 128$
σ	$\sigma/2.8$	$\sigma/4$	$\sigma/8$	$\sigma/11.3$

Further reading on discussed topics

- Sections 3.1-3.3 of Szeliski
- Chapter 3 of Gonzalez and Woods 2002

Acknowledgement

- Some images drawn from the mentioned resources